PREDICTING RISK OF EXPOSURE TO PEAK CONCENTRATIONS IN FLUCTUATING PLUMES

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by

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TABLE OF CONTENTS

	page
Acknowledgements	i
Abstract	ii
Chapter 1 - Introduction	1
Previous Studies of Pipe Rupture Risk	2
Scope of Present Study	3
Chapter 2 - Mean and Variance	5
Mean Concentration for an Elevated Source	5
Mean Concentrations for a Ground Level Source	8
Concentration Fluctuation from an Elevated Source	9
Variance Source Strength q	12
Chapter 3 - Conditional Averages and Intermittency	24
Relating "Total" and "Plume" Statistics	25
Predicting Plume Variance c'^2_q	27
Along-Wind Variation of Plume Intensity, i_p	28
Dissipation of "Plume" Functions Near the Ground	30
Predicting Plume Intermittency	32
Crosswind and Vertical Profiles of Fluctuation Statistics	33
Other Studies of Concentration Fluctuation Statistics	36
Previous Models and Measurements for Intermittency	40
Summary	42

TABLE OF CONTENTS (con't)

	page
Chapter 4 - Probability Distribution and Risk of Exposure	54
Accounting for Plume Intermittency	54
Choosing a Distribution Function	56
Exponential Probability Density	58
Log-Normal Probability Density	60
Observed Probability Densities	61
Observed Cumulative Distributions	64
Core-Bulk Structure in Plume Mixing	67
Chapter 5 - Crosswind Variation of Exposure Risk	78
Crosswind Normalization of Marginal Probability	78
Release Conditions for Sample Calculations	79
Calculating Risk Probability	80
Some Unresolved Difficulties	83
References	88

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ABSTRACT

Existing models to predict concentration fluctuations in plumes are reviewed. These models are found to be inadequate to estimate the risk of exposure to a peak concentration because they do not deal with the intermittent nature of concentration in a meandering plume. A more general theory is developed which accounts for intermittency and predicts the spatial variation of fluctuation intensity and intermittency. Wind tunnel data is used to adjust and verify the theory. Full scale measurements and other laboratory data generally supports the proposed model. Various probability density functions are examined and the log-normal is shown to be the most realistic approximation, based on both full scale and wind tunnel data. Sample calculations of risk of exposure are presented to demonstrate the along-wind and cross-wind changes in risk with varying release height.

CHAPTER ONE

Introduction

Predicting the behavior of a toxic gas plume from a pipeline rupture is one of the most complex and difficult problems in atmospheric diffusion. The rate of gas release changes with time, decreasing as the pipeline is depleted. Initially, the high pipeline pressure makes the exit jet from the rupture supersonic, and causes significant changes in the gas temperature and density. At the same time, the plume rise and atmospheric diffusion occurs in the highly inhomogeneous turbulent boundary layer near the ground. This makes the plume particularly susceptible to changes in terrain, downwind obstacles, and atmospheric stability effects. As a further complication we must deal with the time varying fluctuating concentration rather than the mean value in order to assess the risk of momentarily exceeding some specified harmful threshold level of toxic gas concentration.

The only saving grace in this messy problem is that the entire release is completed typically in a time of 30 minutes or less, with the most dangerous portion at the high initial release rate during the first 60 seconds after rupture. Thus, long term changes in atmospheric conditions such as stability and wind speed need not be considered.

Predicting the risk of toxic gas exposure from a transient release requires us to specify or estimate the following elements that will form a complete model of the process:

1. The release rate of toxic gas from the rupture, and its variation with time.
2. The plume rise due to bouyancy and upward momentum of the gas jet.
3. Turbulent atmospheric diffusion in the along-wind (x) direction that tends to smear out the time varying release rate from the rupture.

4. The vertical and cross wind atmospheric diffusion needed to predict the mean concentration and the variance about this mean of concentration fluctuations at a fixed receptor.

5. The fraction of time (called the intermittency γ) that the plume will be present at the receptor. This appearance and disappearance of the plume with time is due to meandering of the plume axis caused by large scale turbulent motions.

6. The appropriate form of the probability distribution function for the fluctuating concentration. This distribution is required so that the mean concentration, its variance, and the intermittency can be combined to predict the probability of exposure.

7. The sheltering effect of having the receptor indoors, behind a barrier, or in a vehicle as the plume of toxic gas passes overhead.

Previous Studies of Pipe Rupture Risk

Wilson (1979a) developed a series of theoretical models to predict the release and dispersion of toxic gas from high pressure pipeline ruptures. This study dealt briefly with each of the seven elements listed above. However, lack of experimental data, and the need to produce a workable model in short time required that several arbitrary assumptions and a few educated guesses be made regarding the plume behavior.

In a subsequent study Wilson (1981a) made use of experimental data from a series of full scale pipe rupture experiments carried out by the Alberta Petroleum Industry - Government Environmental Committee to refine the model of release rate and plume rise. These experiments verified that the double exponential release rate approximation was reasonable. New estimates were developed for supersonic jet expansion outside the rupture, and the effect of crater drag on jet momentum. These were combined with estimates for entrainment rates of air into the plume in order to define plume trajectory and final rise, which was in reasonable agreement with experimental observations. In addition, Wilson (1981b) published a more complete version of the theory

for along-wind diffusion to predict the ensemble mean concentration in the leading edge of the suddenly started plume release.

Baynes and Scott (1977) carried out a theoretical study of plume fluctuations to predict risk probabilities for H_2S exposure from a continuous source. Suprisingly they found only a weak (factor of 2 or less) dependence of risk of lethal exposure on atmospheric stability and time of exposure, which ranged from 1 to 10 min. However the risk was strongly dependant on the concentration selected as the toxic limit. This result is quite different from the study by Choukalos (1980) who carried out sample calculations using Wilson's (1979) model and found risk levels roughly proportional to the toxic limit selected.

Scope of Present Study

In this study we will focus on three of the elements (4, 5, 6) necessary to estimate the risk of exposure. These elements all share the common theme of investigating the characteristics of concentration fluctuations due to inhomogeneous mixing in order to estimate the probability of exposure to a given concentration. The variables we seek here are, for a fixed receptor location:

1. The mean plume concentration $\overline{c}$.
2. The variance $\overline{c'^2}$ about the mean.
3. The fraction of time the toxic source material is present, which we call γ, the intermittency.
4. The probability density function required to estimate the risk of exceeding a given concentration. This probability density requires the preceeding three variables of mean, variance and intermittency.

An improved estimate for these parameters is possible because of the availability of new experimental data. These measurements of the fluctuating plume concentration statistics were made by Fackrell and Robins (1980a, 1980b)

at the Marchwood Engineering Laboratories of the Central Electricity Generating Board in Great Britain. Analytical models based on this data were developed by Wilson, Robins and Fackrell (1982) for a ground level source, and in a companion paper by Wilson, Fackrell and Robins (1983) for an elevated source. In this study we will refine and extend these models to predict the variables necessary for evaluating the concentration probability density function.

CHAPTER 2

Mean and Variance

Our first task in defining the probability of exceeding some peak concentration is to determine how the mean concentration and its variance change with position downwind from a point source. Because the plume from a pipeline rupture is a transient with time varying release rate, the mean concentration $\bar{c}$ at a fixed downwind receptor is the ensemble average over many identical releases observed at the same time t after rupture. However, all the data we will use to support the theories are for a continuous steady point source release, where the time average mean and the ensemble mean are identical. We will use $\bar{c}$ to indicate both of these types of averages.

Mean Concentration for an Elevated Source

For an elevated source, Gaussian dispersion models are popular because of their simplicity, combined with their ability to simulate most of the physical features of atmospheric dispersion. However, the Gaussian model is only strictly valid for dispersion in homogeneous turbulence, and we will see later that it does have some deficiencies when estimating dispersion near the ground, where both the intensity and scale of turbulence vary significantly with height.

Considering the uncertainties in estimating plume rise and atmospheric stability, the Gaussian plume model is a reasonable approximation for the mean concentration. With ground level reflection the mean is given by

$$\bar{c} = \frac{Q}{2\pi U_c \sigma_y \sigma_z} G_m\left(\frac{y}{\sigma_y}\right) F_m\left(\frac{z}{\sigma_z}\right) \qquad (2\text{-}1)$$

$$G_m\left(\frac{y}{\sigma_y}\right) = \exp\left(-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right) \qquad (2\text{-}2)$$

$$F_m\left(\frac{z}{\sigma_z}\right) = \exp\left(\frac{-1}{2}\left(\frac{z-h}{\sigma_z}\right)^2\right) + \exp\left(\frac{-1}{2}\left(\frac{z+h}{\sigma_z}\right)^2\right) \qquad (2\text{-}3)$$

In some cases where experimental data is being analyzed it is more convenient to specify the plume dimensions by the half-widths δ_y and δ_z, based on the distance to the point where the mean concentration is reduced to half the value on the plume axis with no ground level reflection effects. The half width does not require an integral across the profile as do the standard deviations σ_y and σ_z, and are therefore easier to determine. For a Gaussian profile the half widths and standard deviations are related by the simple equation

$$\delta_y = \sqrt{2A}\ \sigma_y \qquad (2\text{-}4)$$

$$\delta_z = \sqrt{2A}\ \sigma_z \qquad (2\text{-}5)$$

where $A = \ln(2) = 0.693$, so that $\delta = 1.177\,\sigma$

For a source height far above ground, the wind speed at source height is a reasonable estimate for the plume convection velocity U_c. However, this rule of thumb does not apply to low level sources, and for a ground level source it completely degenerates, predicting $U_c = 0$. The problem of specifying the plume convection velocity for low level sources is discussed in detail by Wilson (1981b), who uses the "exact" Lagrangian similarity solution to fix the height z_c at which to evaluate the convection velocity

$$z_c = h + 0.17\,\sigma_z \qquad (2\text{-}6)$$

For a power law variation of vertical velocity the convection velocity is related to the wind speed at some reference height z_{ref} by

$$U_c = U_{ref} \left(\frac{z_c}{z_{ref}} \right)^n \qquad (2\text{-}7)$$

where n is the power law exponent, which depends on surface roughness and atmospheric stability.

Variability of atmospheric conditions with time, as well as the expense and difficulty of traversing vertically through a plume makes it very difficult to verify this Gaussian model for mean concentration. As an alternative, wind tunnel simulations in small scale have the necessary precision to test boundary layer diffusion models. The data from one such simulation of Fackrell and Robins (1980b) is shown in Figures 1 and 2 for vertical and cross-wind profiles. For these experiments the neutrally stable wind tunnel boundary layer was modelled in about 500:1 scale. Assuming a full scale boundary layer height of 650 meters, the elevated source in this simulated layer was located at an equivalent full scale height of about 125 meters above ground. The results in Figure 1 show that a Gaussian profile is an excellent approximation for cross wind diffusion, as confirmed by full scale measurements reported by Pasquill (1974, p. 175, 205). As expected, Figure 2 shows the Gaussian diffusion model over-predicts ground level concentrations because it ignores the decreasing turbulent diffusivity near ground level, and instead assumes a constant turbulent diffusion coefficient across the entire boundary layer.

In practice, full scale atmospheric diffusion experiments seldom determine σ_z directly, but instead compute it from measurements of ground level concentration.

This automatically adjusts the vertical diffusion coefficient in the Gaussian model to accurately match ground level concentrations. In the Gaussian form given here, (2-6) compensates somewhat for the tendency of the model to over-predict, by gradually increasing the height at which the convection velocity is determined. This increased height produces a gradually increasing convection velocity, which in turn lowers predicted plume concentrations.

Mean Concentrations for a Ground Level Source

For a ground level source, a Gaussian profile is a good description of cross-wind spread in both full scale, Pasquill (1974, p. 175, 205) and for wind tunnel boundary layers, Robins (1978). The vertical concentration distribution for ground level source is influenced by the variation of mean velocity and turbulence intensity and scale with height above ground. Vertical profiles are well represented by the form

$$\overline{c} \propto \exp\left[-A\left(\frac{z}{\delta_z}\right)^a\right] \qquad (2\text{-}8)$$

where $A = \ln(2) = 0.693$, and δ_z is the profile half width. Wind tunnel data of Robins (1978) is almost perfectly fitted using $a = 1.7$, while Pasquill (1974, p. 175, 205) finds $a = 1.5$ from various full scale experiments.

What is most relevant however, is that Pasquill (1974, p. 350) shows that varying the exponent in the range $1.5 < a < 2.0$ results in only a 15% effect on ground level concentration. This is much less than most other uncertainties in atmospheric diffusion, and justifies the use of the Gaussian diffusion model of (2-3), where $a \equiv 2.0$, for all source heights. The applicability of a single functional form for the mean concentration field at all source heights is particularly important for analysis of dispersion from pipeline ruptures.

Because the release rate from a rupture decreases with time, the plume rise will also decrease, causing the initial phase of release to behave as an elevated source, and the final phase to act like a ground level source. It would be inconvenient to have to specify a set of diffusion equations whose functional form varied with time.

Concentration Fluctuations from an Elevated Source

For the case of a pipeline rupture, the mean concentration varies with time as the release rate decreases during depletion of the line. Because the plume is a transient, the mean and the variance about this mean are the ensemble averages of many identical releases. However, the only data available to test theoretical models is for continuous steady sources where the ensemble mean and variance are the same as the time averaged values at the same downwind location. Only these time mean values will be discussed and the reader should keep in mind that these are analogous to the ensemble mean values for the pipe rupture.

In the discussion that follows we refer to the "total" variance of concentration fluctuations. The total variance includes departures from the mean concentration that occur when the plume meanders away from a receptor reducing the concentration to zero. This total variance will always be larger than the "conditional" variance which is measured only when the plume material is present and the concentration is greater than zero. In the next chapter we will discuss these conditionally averaged values, and show that they are more useful than the total variance in estimating the probability of exceeding some peak concentration. For the moment, we will focus on the total variance simply because existing models for concentration fluctuations deal only with this parameter. The concentration fluctuation c' is defined as the departure from the time (ensemble) mean concentration $\overline{c}$, which is computed using all observed concentrations over the specified time interval, including zero values.

Wilson, Fackrell and Robins (1983) developed an approximate model for the total variance in the plume from a point source. Briefly, the distribution of total variance was represented by a point source diffusion process across the plume for crosswind and vertical profiles, and by an along-wind dissipation process to determine the effective strength of the variance source. This model was inspired by Csanady's (1973) theory which made use of a variance source, and Netterville's (1979) wind tunnel observations of a diffusion-like behavior of crosswind variance profiles. Following Netterville's suggestion, the effect of increased variance dissipation near ground level was modelled by an image sink for variance, much like the image source term used to account for ground plane reflection in the Gaussian mean concentration model.

Measurements in the plume from a ground level source led Wilson, Robins and Fackrell (1982) to conclude that the effective sources of variance should be located away from the plume axis near the points of maximum $\overline{c'^2}$ production. The final model used a pair of variance sources located at $y = \pm \beta\delta_y$, one on each side of the plume axis, and an equal distance above the actual source height. Transforming their equations to a format using σ_y, σ_z rather than δ_y, δ_z, the total variance, including fluctuations caused by intermittency, is

$$\overline{c'^2} = \left(\frac{q}{2\pi U_c \sigma_y \sigma_z}\right)^2 G_v\left(\frac{y}{\sigma_y}\right) F_v\left(\frac{z}{\sigma_z}\right) \qquad (2\text{-}9)$$

$$G_v\left(\frac{y}{\sigma_y}\right) = \frac{1}{2}\exp\left(-\frac{1}{2}\left(\frac{y}{\sigma_y} - \phi\right)^2\right) + \frac{1}{2}\exp\left(-\frac{1}{2}\left(\frac{y}{\sigma_y} + \phi\right)^2\right) \qquad (2\text{-}10)$$

$$F_v\left(\frac{z}{\sigma_z}\right) = \exp\left[-\frac{1}{2}\left(\frac{z-h_v}{\sigma_z}\right)^2\right] - \alpha\exp\left[-\frac{1}{2}\left(\frac{z+h_v}{\sigma_z}\right)^2\right] \qquad (2\text{-}11)$$

with the variance sources a distance $y = \pm \phi\sigma_y$ on each side of the plume axis and at a height h_v above ground. This height for the pair of variance sources is given by

$$\frac{h_v}{\sigma_z} = \left[\left(\frac{h}{\sigma_z} \right)^2 + \phi^2 \right]^{0.5} \qquad (2\text{-}12)$$

The parameter ϕ which specifies the location of the pair of variance sources is related to the original parameter β used by Wilson, Robins and Fackrell (1982) by the simple transformation

$$\phi = \sqrt{2A}\ \beta \qquad (2\text{-}13)$$

With their choice of $\beta = 0.60$ and the definition $A \equiv \ln(2) = 0.693$ we find $\phi = 0.706$.

A comparison of the modified Gaussian theory for variance is shown in Figures 3 and 4. The agreement with observations is remarkably good, considering that the model describes the variance profiles with the same σ_y and σ_z determined for the mean concentration profiles. In Figures 3 and 4, both the theoretical and experimental profiles are normalized at each downwind position with the maximum variance $\hat{c}^2$ in the y=0 plane.

A lack of data near ground level makes it difficult to specify the sink strength α for the elevated source in Figure 3 and 4. This is clearly evident in Figure 5 where the vertical profile at 4.79 boundary layer thickness downwind (about 3 km in full scale) is compared to different choices for α in (2-11).

It is very important to make an accurate estimate for the sink strength α, because this factor strongly influences the intensity of concentration fluctuations near ground level, and therefore the risk of exposure to high momentary peak

concentrations. The choice of $\alpha = 0.6$ is not simply a compromise between a totally dissipating surface with $\alpha = 1.0$ and a passive surface with $\alpha = 0$, but instead was chosen using the measurements Robins and Fackrell (1979) in a ground level source. These vertical fluctuation profiles provided measurements much closer to the surface than did the elevated source, as shown in Figure 6, and justify our choice of

$$\begin{bmatrix} \alpha = 0.60 \\ \phi = 0.706 \quad (\beta = 0.60) \end{bmatrix}$$

Variance Source Strength q

Unlike the mass release rate Q which remains constant at each downwind plume cross section, as long as there is no chemical reaction or surface deposition, the variance "sources" are depleted by the process of molecular diffusion. Far downwind molecular diffusion has had enough time to smear out all concentration fluctuations reducing the variance to zero. At the other extreme, close to the source, plume meandering and entrainment of ambient air causes an increase in variance with downwind distance, a process we label as "production".

Wilson, Robins and Fackrell (1982) modelled this production and dissipation process with a semi-empirical balance for the variance fluxes. This resulting equation for the total variance source strength q for an elevated release at height h is, from Wilson, Fackrell and Robins (1983)

$$\frac{q}{Q} = \frac{0.38\,\lambda}{\lambda^2 + \lambda_0^2} \qquad (2\text{-}14)$$

where

$$\lambda = \sqrt{\frac{\sigma_y \sigma_z}{H}} \qquad (2\text{-}15)$$

and $$\lambda_o = \frac{0.119\ d}{0.033\ z_e + d} \qquad (2\text{-}16)$$

All the above equations have been transformed from their original form where they were expressed in terms of $\xi \equiv \sqrt{\delta_y \delta_x}/H$ by using $\xi = 1.177\ \lambda$, derived from (2-4) and (2-5).

The use of the parameters λ or ξ to specify downwind distance is based on the expectation that most of the effects of varying turbulence scale under different atmospheric stabilities and terrain roughness will be accounted for by specifying downwind position in terms of plume dimensions rather than distance from the source. While there is nothing in the model for variance source strength to prohibit its application to varying surface roughness and atmospheric stability, it must be kept in mind that the empirical constants in (2-14) and (2-16) were found for a neutrally stable boundary layer with a surface roughness of $z_o/H = 4 \times 10^{-4}$ (about 25 cm in full scale).

The virtual origin λ_o in (2-16) contains all the effects of source size and atmospheric turbulence scale. Because turbulence scale increases with increasing height, z, in a boundary layer, some scale height, z_e, must be specified that reflects the average turbulence scale seen by the plume. In their model, Wilson, Fackrell and Robins (1983) chose the source height to represent turbulence scales, setting $z_e = h$. However, as the plume from an elevated source moves downwind it will sample the small turbulence scales near the ground, and large scales above the source height. The wind tunnel data shows that the sampling of small scales is the dominant effect as the plume moves downwind, and that the use of a constant scale $z_e = h$ causes (2-14) to overpredict the variance source strength q far downwind. Several alternate functions for z_e were tested, in order to produce a

scale height which decreased with downwind distance to account for decreasing effective scale. One simple function that improved that data correlation was

$$z_e = h \exp\left[-\left(\frac{\sigma_z}{h}\right)^2\right] \tag{2-17}$$

and we will adopt this form here.

The most useful parameter for specifying the level of concentration fluctuation is the intensity, i, which may be found from (2-1) and (2-9) as

$$i \equiv \frac{\sqrt{\overline{c'^2}}}{\overline{c}} = \left(\frac{q}{Q}\right) \frac{\left[G_v\left(\frac{y}{\sigma_y}\right) F_v\left(\frac{z}{\sigma_y}\right)\right]^{0.5}}{G_m\left(\frac{y}{\sigma_y}\right) F_m\left(\frac{z}{\sigma_z}\right)} \tag{2-18}$$

with the G and F functions from (2-2), (2-3), (2-10) and (2-11).

Comparison of the semi-empirical model for source strength given by equations (2-14), (2-16), and (2-17) is shown in Figure 7. Using equation (2-17) for the scale height gives a good estimate for fluctuation intensity computed from (2-18). Near the source, Figure 7 shows that the total fluctuation intensity, including intermittent zero values, is strongly dependent on source diameter. In the next chapter, where we consider conditional averages when only the plume material is present, we will see that these high fluctuation intensities are caused entirely by plume meandering, which induces long periods of zero concentration for small source sizes.

As expected, with four adjusted constants and a variable scale height, the semi-empirical model gives good agreement with the observations of total concentration variance on which it was based. We will show later that this model is not simply an exercise in curve fitting, but has sufficient physical justification to be useful in predicting the conditionally averaged values in the plume. Another

way to test this semi-empirical model for source strength is to apply it to a different set of measurements. A good test of the equation for source strength is to use it to predict the maximum variance $\hat{c}^2$ for a ground level source. For this case, the data of Wilson, Robins and Fackrell (1982) is compared in Figure 8 with calculated values from the model extrapolated to zero source height. The model agrees remarkably well with the data in this asymptotic limit. The data shown is for two different source sizes, and additional measurements over a wide range of source sizes from $2.5 \times 10^{-3} < d/H < 3.1 \times 10^{-3}$ showed that the observed total variance for a ground level source is independent of source diameter. This result suggests that there is little or no plume meandering for a ground level source diffusing in the small scale turbulence near the surface. The semi-empirical model in (2-14) also becomes independent of source diameter in the asymptotic limit as the source height (and therefore the scale height z_e) goes to zero in (2-16).

In summary, models have been developed and tested for predicting the mean and total variance fields downwind from either a ground level or elevated source. While such models are useful, they fall short of our objective of providing the necessary variables to define the probability distribution function that will permit us to estimate the probability of exceeding some peak concentration. To properly define these probability densities we must separate the effect of plume meandering, which generates zero concentration values, from actual concentration fluctuations within the plume material caused by inhomogeneous turbulent mixing.

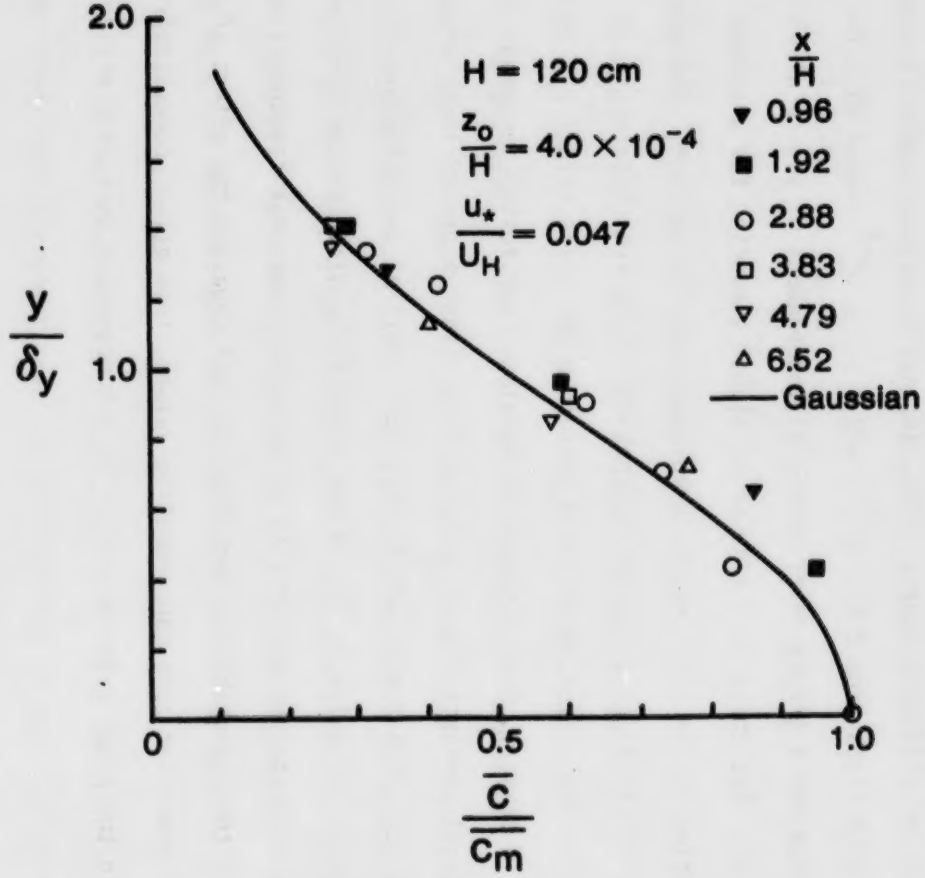

Figure 1. Crosswind mean concentration profiles from an elevated 0.85 cm diameter point source at h/H = 0.192 in a neutrally stable wind tunnel boundary layer of thickness H. data from Fackrell and Robins (1980b).

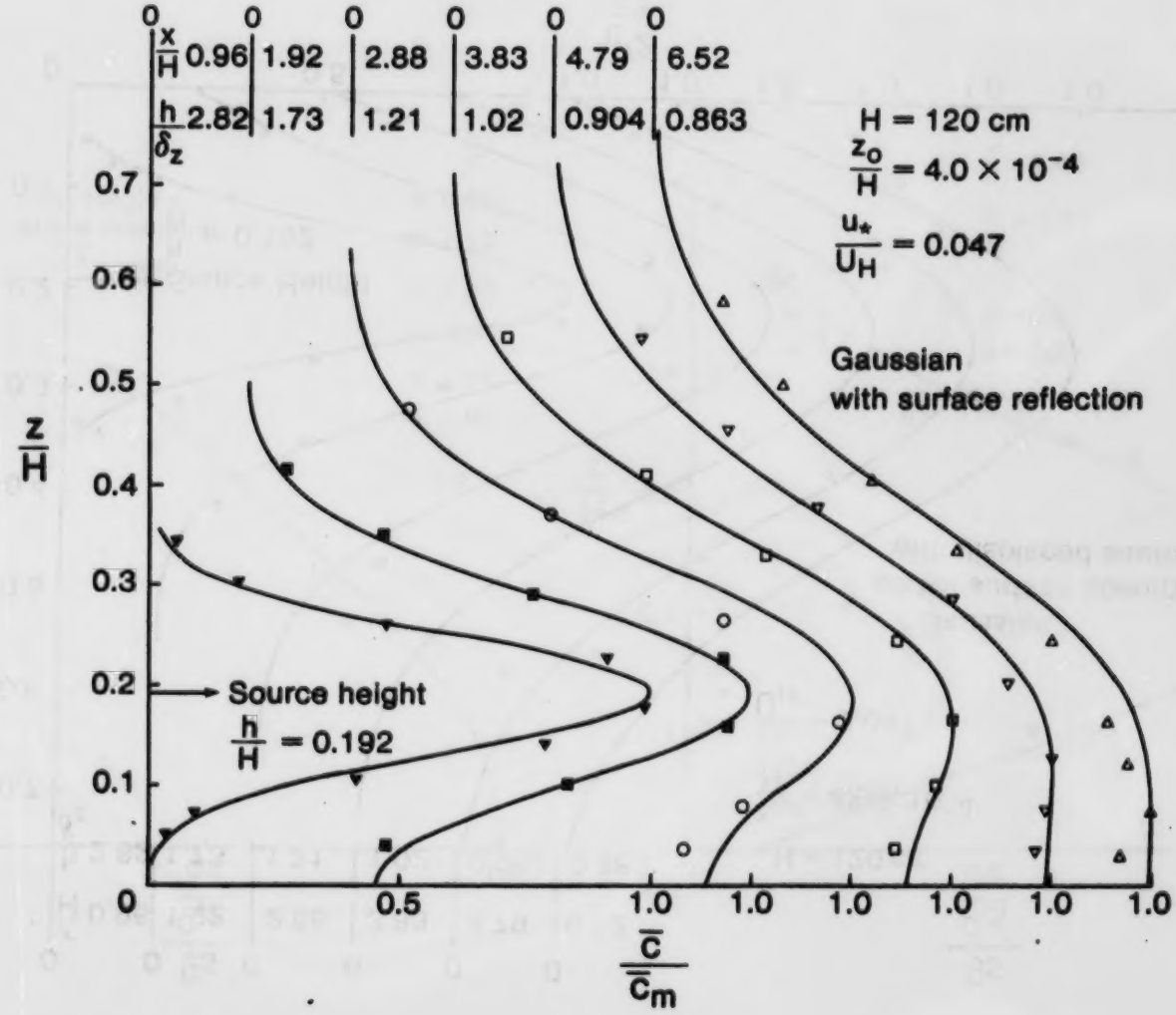

Figure 2. Vertical mean concentration profiles from a 0.85 cm diameter source in a neutrally stable wind tunnel boundary layer of thickness H. Data from Fackrell and Robins (1980b).

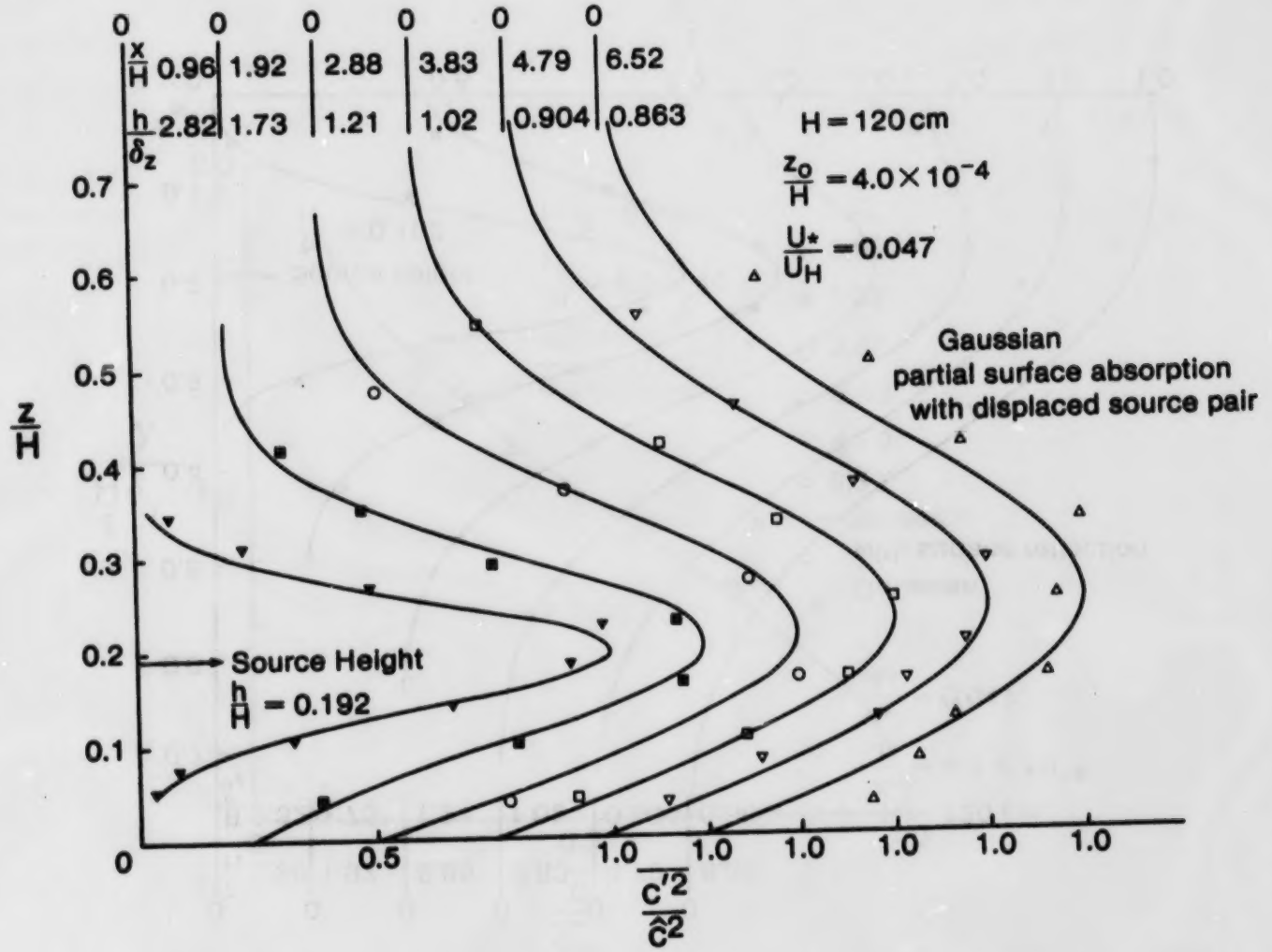

Figure 3. Total plume variance profiles from a 0.85 cm diameter source in a neutrally stable wind tunnel boundary layer of thickness H.
From Wilson, Fackrell and Robins (1983) with $\alpha = 0.6$, $\beta = 0.6$.

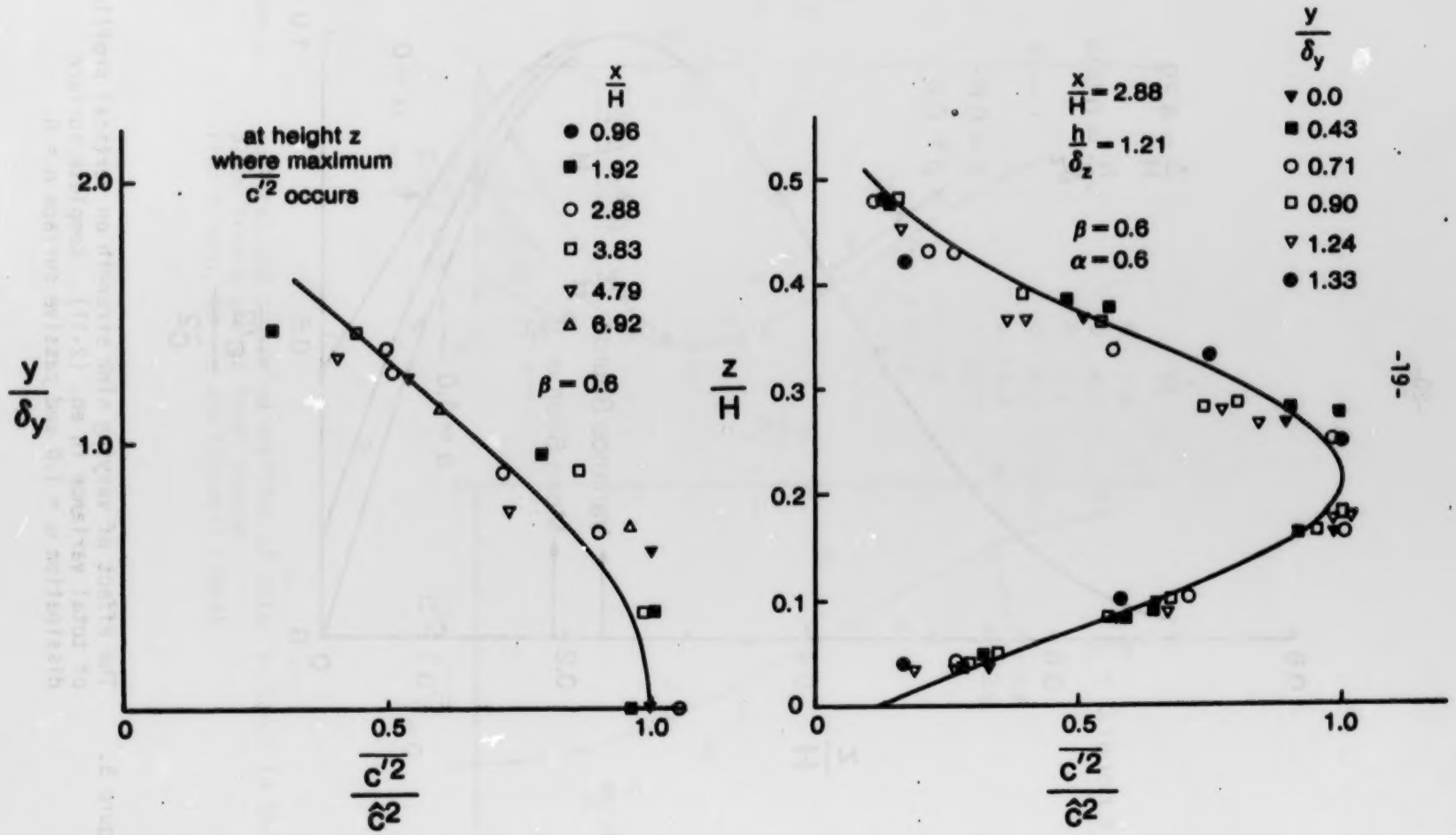

Figure 4. Crosswind profiles of total variance for an elevated plume with h = 0.192 H in a neutrally stable wind tunnel boundary layer of thickness H. From Wilson, Fackrell and Robins (1983).

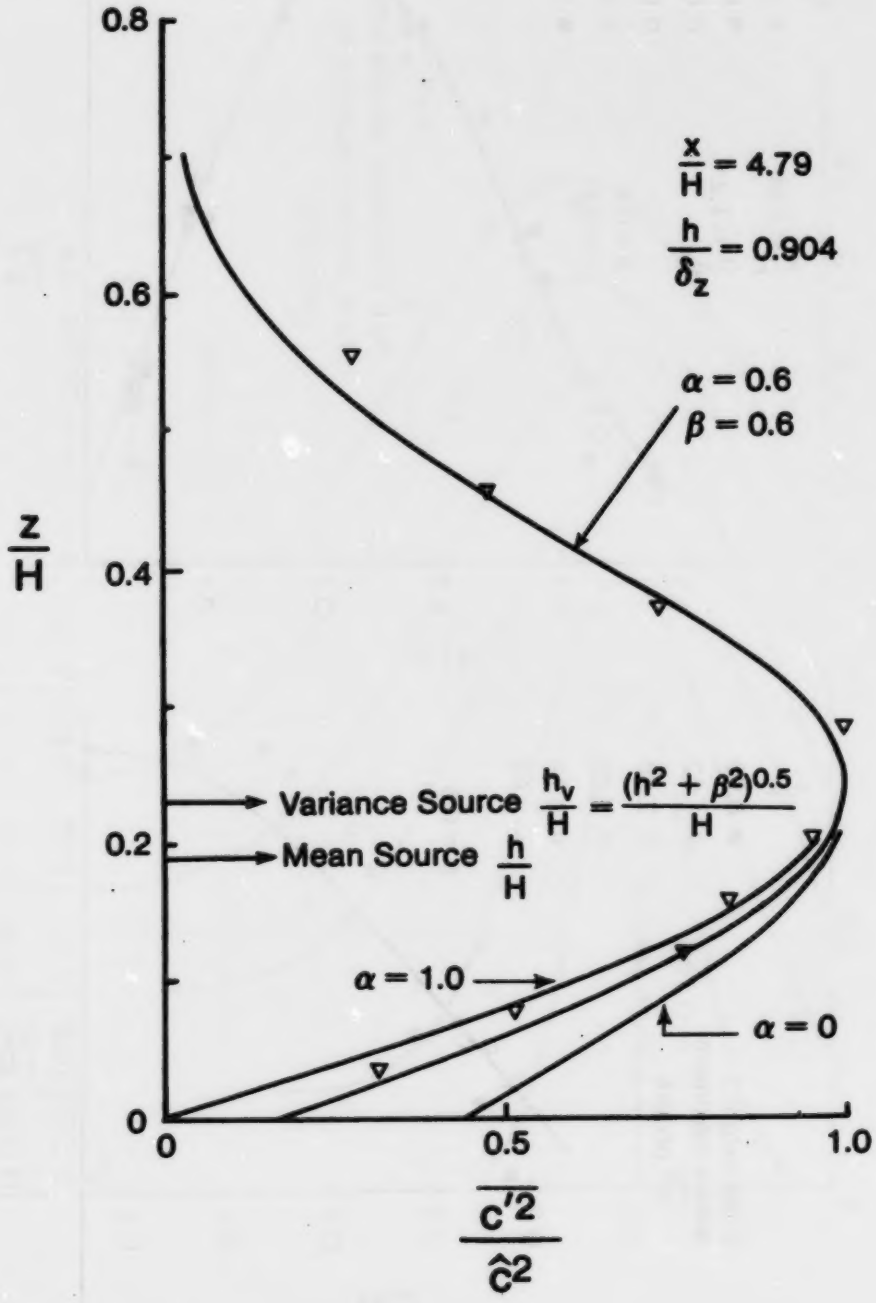

Figure 5. The effect of varying sink strength on vertical profiles of total variance in eq. (2-11). Complete surface dissipation $\alpha = 1.0$ and passive surface $\alpha = 0$.

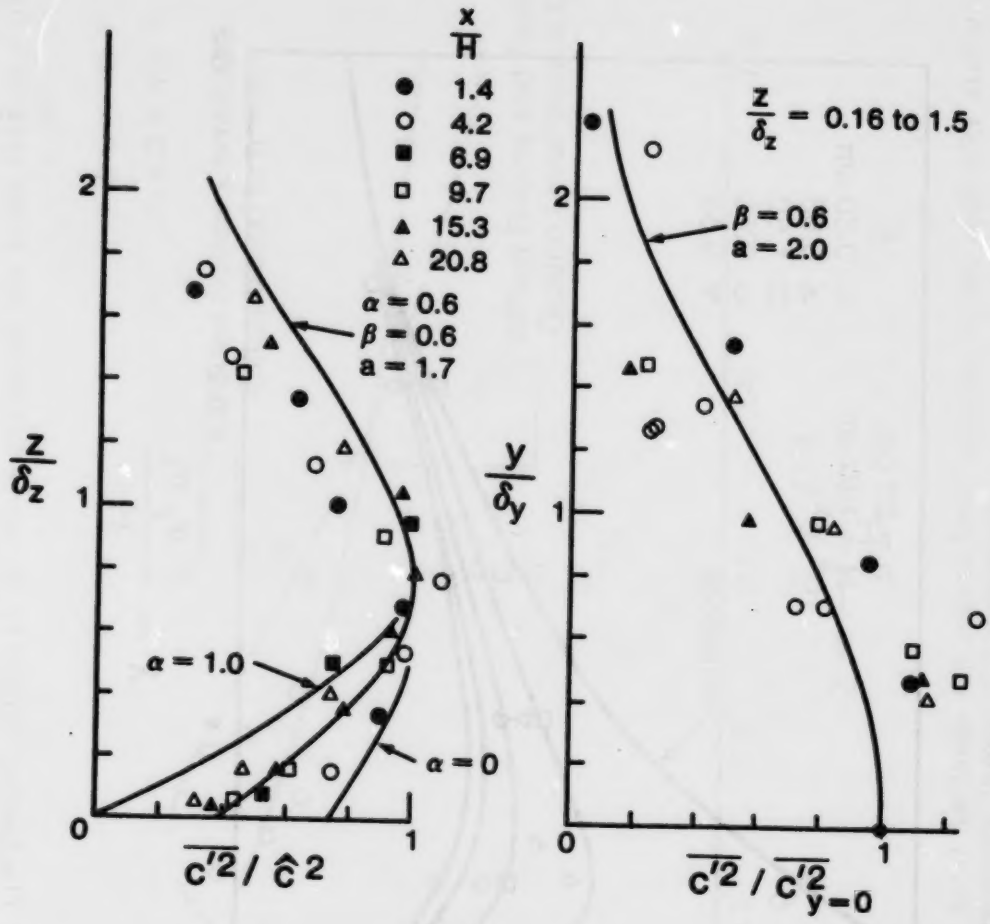

Figure 6. Vertical and crosswind profiles of total variance in the plume from a ground level source. From Wilson, Robins and Fackrell (1982).

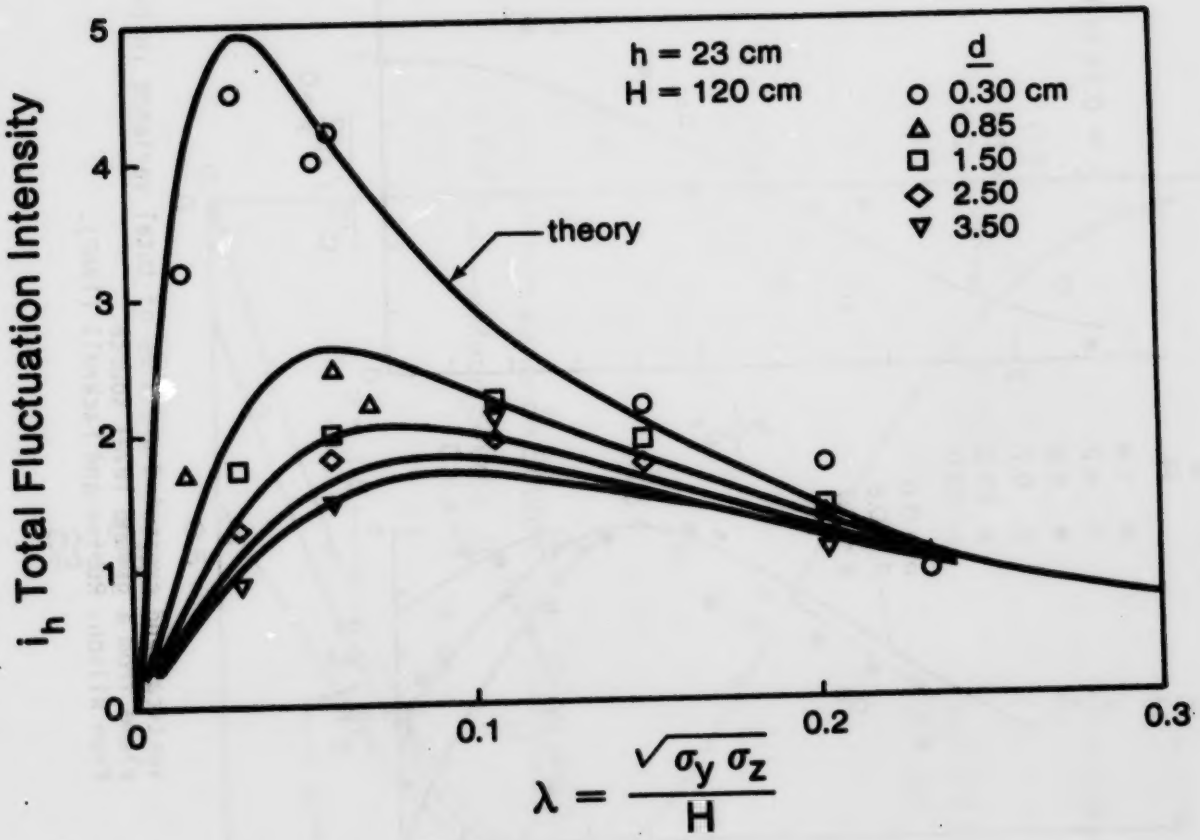

Figure 7. Total fluctuation intensity at source height on the plume axis (y = 0) for varying source size. Elevated plume in a neutrally stable wind tunnel boundary layer of thickness H.
Data of Fackrell and Robins (1980a).

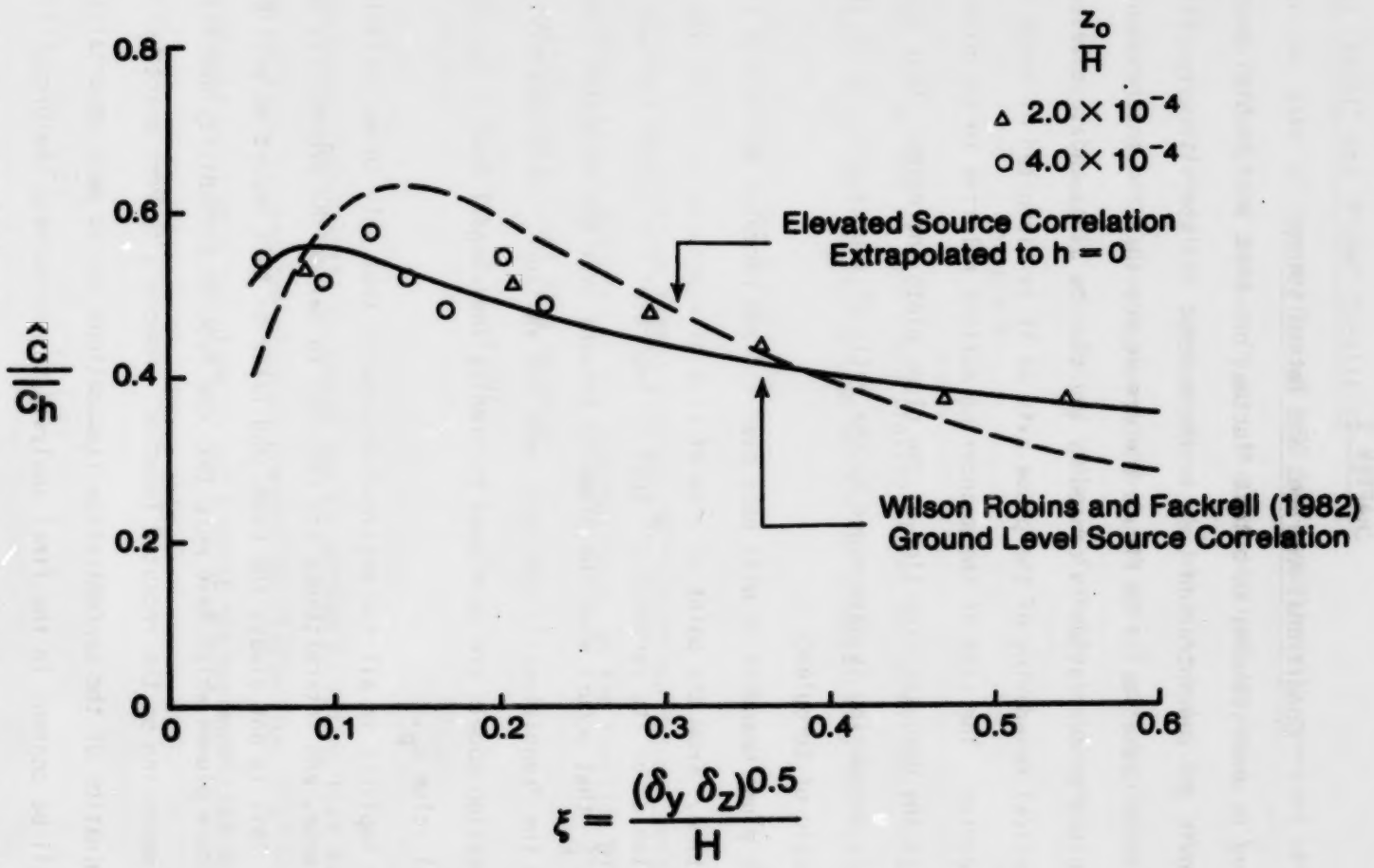

Figure 8. Maximum total variance for ground level sources in two neutrally stable wind tunnel boundary layers of thickness H.
O H = 120 cm, d = 1.5 cm, Δ H = 36 cm, d = 1.1 cm
From Wilson, Fackrell and Robins (1983).

CHAPTER 3

Conditional Averages and Intermittency

The key to understanding turbulent fluctuations that lead to high momentary peaks of toxic gas concentration lies in the concept of intermittency. This effect is shown dramatically in Figure 9 where we see that the intermittent nature of plume concentration is caused by the side to side, and to a lesser extent, vertical meandering of the plume axis as it is distorted by large scale turbulent eddies. The size of the meandering motions relative to the plume width decreases in the downwind direction as the plume width increases. Near the source, this meandering is determined by the ratio of source size, d, to the integral scale of turbulence Λ.

As the plume meanders it will pass over a fixed receptor for only a fraction, γ, of the time. From the point of view of risk of exposure, it is not the mean concentration $\bar{c}$ or its variance $\overline{c'^2}$ that is important but rather the mean $\bar{c}_p$ and variance $\overline{c'^2_p}$ that occurs when the plume is present over the receptor. These are called the "conditionally averaged" mean and variance. Unfortunately, most plume dispersion models are developed to predict the overall mean $\bar{c}$ and not the conditional value $\bar{c}_p$.

It is implicit in all the previous discussion that only three variables, the mean, variance, and intermittency are required to define the probability density function. This is not always the case, and later in this report we will examine dual structure plumes which have more than one type of probability density function, depending on the receptor location. However, in most practical problems the determination of the concentration fluctuations has so many uncertainties, that we will be content in the final analysis to approximate the probability density with a simple function that needs only the specification of mean, variance and intermittency.

Relating "Total" and "Plume" Statistics

For the sake of brevity let us call the conditionally averaged variance when source material is present (i.e. $c > o$) the "plume" variance, $\overline{c'^2_p}$. This will distinguish it from the total variance $\overline{c'^2}$ which includes the intermittent periods when $c = 0$. The fluctuation c' about the mean $\bar{c}$ used to define this total variance is found by subtracting the mean from the instantaneous concentration

$$c' = c - \bar{c} \quad \text{for } c \geq 0 \tag{3-1}$$

Censoring the periods of zero concentration allows the conditional "plume" fluctuation to be defined in the same way, shown schematically in Figure 9.

$$c'_p = c - \bar{c}_p \quad \text{for } c > 0 \tag{3-2}$$

Becker, Hottel and Williams (1965) related the "total" and "plume" mean and variance statistics. Defining γ, the intermittency, as the fraction of time source material is present at a receptor

$$\bar{c} = \gamma \bar{c}_p + (1-\gamma)(0)$$

$$\bar{c}_p = \frac{\bar{c}}{\gamma} \tag{3-3}$$

The relationship between the variances may be derived by noting that during the fraction of time, γ, that the plume is present the fluctuation is $c' = c_p - \bar{c}$. For the period $(1-\gamma)$ when the plume is not present the concentration must be zero, and $c' = -\bar{c}$ (see Figure 9). Using this,

$$\overline{c'^2} = \gamma \overline{(c_p - \bar{c})^2} + (1 - \gamma) \overline{(-\bar{c})^2} \tag{3-4}$$

Expanding the squared terms and using (3-3) leads to

$$\overline{c'^2} = \left(\frac{1-\gamma}{\gamma}\right) \bar{c}^2 + \gamma \overline{c_p'^2} \tag{3-5}$$

A particularly useful form of (3-5) results when we divide by $\bar{c}^2$ to form the "total" and "plume" fluctuation intensities $i^2 = \overline{c'^2}/\bar{c}^2$ and $i_p^2 = \overline{c'^2}/\bar{c}^2$

$$i^2 = \frac{i_p^2}{\gamma} + \left(\frac{1-\gamma}{\gamma}\right) \tag{3-6}$$

This result shows that even when there are no concentration fluctuations in the plume material (i.e. $i_p \equiv 0$) there will still be a total fluctuation intensity of

$$i = \left(\frac{1-\gamma}{\gamma}\right)^{0.5} \tag{3-7}$$

caused by having the mean concentration pulled down to zero by intermittency.

What is more important is that the fluctuations produced by intermittency do not contribute to the risk of exposure to high peak concentrations. To illustrate this point consider two plumes with the same observed fluctuation intensity, i. One plume has no inhomogeneous mixing fluctuations but is intermittent, while the other has no intermittency. These two situations are presented schematically in Figure 10, which shows that the fluctuations due to intermittency make no contribution to peak levels because intermittency can only act to pull the concentration down.

Clearly, a method for separating fluctuations caused by plume mixing and by intermittency is required before we can estimate the risk of exposure to concentration peaks. In other words, what is needed is a model for $\bar{c}_p$, $\overline{c_p'^2}$ and γ not for the total mean and variance $\bar{c}$ and $\overline{c'^2}$.

Predicting Plume Variance $\overline{c_p'^2}$

Because we have already developed and adopted semi-empirical equations for the total mean $\bar{c}$ and variance $\overline{c'^2}$ of a plume, a model for the conditionally averaged "plume" statistics $\bar{c}_p$ and $\overline{c_p'^2}$ is sought which is consistent with the functional forms of these "total" equations. The one new parameter that must be specified is the intermittency, γ, from which (3-3) can be used to immediately calculate $\bar{c}_p$ from the Gaussian plume model for $\bar{c}$ given in (2-1).

The most obvious way to obtain the plume mean and variance is to develop a functional model for the intermittency, γ, and use this in (3-3) and (3-5) to compute the mean and variance in the plume. Several investigators, for example Becker, Hottel and Williams (1965), (1967), have taken this direct approach. However, for plume dispersion it is easier to develop a model for the conditional "plume" variance directly and use the reverse procedure of computing the intermittency from the "total" and "plume" variances. The reason that this reverse procedure works best is because the conditional "plume" intensity i_p remains almost constant across a plume cross section for diffusion in homogeneous turbulence. Fackrell (1978) observed a constant value of i_p across a plume in a wind tunnel model study of the stack plume from a thermal generating plant. This result suggests that the inhomogeneous mixing of source material with entrained air has some self preserving structure that depends on time of travel from the point of release, rather than on a diffusion process across the plume. We will later demonstrate that the assumption of constant plume intensity, i_p leads to realistic predictions for the intermittency factor γ.

What is needed here is some way to separate the intermittency and conditional variance that are combined in our present model for total variance.

A reanalysis of the data of Robins and Fackrell (1979) and Fackrell and Robins (1980a), (1980b) was carried out to extract i_p from observations of $\overline{c}$, $\overline{c'^2}$ and γ. This data was then used to help construct and verify an extended version of the total variance model, which allows conditional values and intermittency to be predicted theoretically. The essential assumptions used to extend the total variance equations to estimate conditional values are:

1. A balance between production of fluctuations from ambient air entrainment into the plume and dissipation by molecular diffusion is used to produce an along-wind variation of i_p.
2. The assumption of constant intensity i_p across a plume cross section far above the influence of inhomogeneous mixing and dissipation near ground level.
3. An extension of the image sink for total variance $\overline{c'^2}$ to account for inhomogeneous mixing and increased dissipation of fluctuations near the ground surface.

Along-Wind Variation of Plume Intensity, i_p

In principle, a semi-empirical model for the conditional intensity i_p requires the measurement of the total mean and variance $\overline{c}$ and $\overline{c'^2}$and the intermittency γ at many points downwind. From this, the intensity, i, can be computed and (3-6) used to recover the conditional "plume" intensity i_p, from which the empirical correlation is developed. Here we will take another approach, by <u>simulating</u> the conditions where $\gamma = 1.0$ and $i_p = i$ in the existing equations which model the total variance. This theoretical extension will allow us to predict "plume" variance, and at the same time assures that the equation for i_p will be physically and mathematically consistent with the existing model for total intensity, i.

Intermittency, γ, at any point is caused by large scale eddies that force the plume to meander. In a turbulent boundary layer, the fraction of eddies that

are large enough to contribute to meandering increases with height above ground, as the integral scale of turbulence Λ increases.

As shown schematically in Figure 9, $\gamma = 1.0$ should be observed on the plume axis when the turbulence scale Λ is small. Fackrell and Robins' (1980a) measurements of the effects of source size indicate that $\gamma = 1.0$ will be observed on the plume axis when $\Lambda < d$. Our key assumption here will be that this condition can be simulated in the total variance model by setting the scaling height $z_e = 0$ in (2-16). This simple condition should make the plume appear to be diffusing in a field of small scale turbulence where $\gamma = 1.0$, and for which (3-5) shows the conditional and total variances will be equal, with $\overline{c'^2} = \overline{c'^2_p}$. With $z_e = 0$, (2-14) and (2-16) may be combined to give the conditional variance source strength

$$\frac{q_p}{Q} = \frac{0.38\ \lambda}{\lambda^2 + (0.119)^2} \tag{3-8}$$

keeping in mind that all of this applies only on the plume axis for an elevated source with $h \gg \sigma_z$. Far above this ground plane influence the plume variance may be computed from the total variance model (2-9), (2-10) and (2-11) by setting $z = h$, $y = 0$ and $h \gg \sigma_z$.

$$\overline{c'^2_{p\infty}} = \left[\frac{q_p}{2\pi\ U_c \sigma_y \sigma_z}\right]^2 \exp\left[\frac{-\phi^2}{2}\right] \tag{3-9}$$

where the subscript "∞" is used to denote this conditional variance on the plume axis far from the dissipative effects of the ground surface.

Noting that $\gamma = 1.0$ for this situation, (3-3) shows that $\overline{c}_p = \overline{c}$, which can be computed from the Gaussian plume model of (2-1), (2-2) and (2-3) to give

$$\bar{c}_{p\infty} = \frac{Q}{2\pi U_c \sigma_y \sigma_z} \tag{3-10}$$

Combining (3-9) and (3-10) yields the conditional fluctuation intensity

$$i_{p\infty} = \frac{q_p}{Q} \exp\left[-\phi^2\right] \tag{3-11}$$

which we will assume is constant over the entire plume cross section at any fixed downwind x (or λ) location.

Dissipation of "Plume" Fluctuations Near the Ground

We will make the simplest assumption; that the dissipation of variance by the small scale turbulence near the ground plane has the same relative effect on the conditional intensity i_p as it does on the total intensity i. That is,

$$\frac{i_p}{i_{p\infty}} = \frac{i}{i_\infty} \tag{3-12}$$

where the subscript "∞" refers to the intensity at the same location but with the ground plane removed. In practical terms the ground plane effect is removed by simply deleting the image source term in (2-3) for the mean, and the image sink term in (2-11) for the variance. If we do this, it can be shown after considerable tedious manipulation that (2-18) and (3-12) result in

$$i_p = i_{p\infty}\left[\frac{1 - \alpha \exp\left(-2\frac{h_y z}{\sigma_z^2}\right)}{1 + \exp\left(-2\frac{h_z}{\sigma_z^2}\right)}\right] \tag{3-13}$$

This completes our model for i_p, with $i_{p\infty}$ computed from (3-8) and (3-11).

How valid is our assumption that the dissipative effects induced by the ground surface are the same for both the conditional and total fluctuation

intensities i and i_p? The reasoning behind this assumption is based on the ground level source data of Robins and Fackrell (1979) used to construct the original model for surface effects on total variance in Wilson, Robins and Fackrell (1982). Their data showed that for a ground level source, the small turbulence scales allowed little meandering and kept $0.97 < \gamma < 1.0$ for $z < 0.7\sigma_z$. Thus, because the total and plume fluctuation intensities are the same when $\gamma = 1.0$, the model for surface effects applies to both i_p and i. In other words, the original model for the total variance near the surface was inadvertently also a model for conditional "plume" variance as well, and the concept of image sinks to simulate surface effects can be transferred directly to conditional statistics.

The theory from (3-8), (3-11) and (3-13) is compared in Figure 11 to measurements in an elevated source in a wind tunnel boundary layer. The plume fluctuation intensity at source height i_{ph} is in excellent agreement with the theory, considering that the theory was developed through physical arguments to recover the conditional "plume" fluctuation variance from the "total" variance which included intermittency effects. The two theoretical curves in Figure 11 show the effect of the small scale turbulence near the surface in dissipating the fluctuation intensity $i_{p\infty}$ far above ground to i_{ph} at source height. For the particular source height chosen for the experiments, the theory predicts that ground surface effects will not be felt until after production by inhomogeneous mixing has maximized the concentration fluctuation intensity. For lower source heights, the surface effects would have caused i_{ph} to differ from $i_{p\infty}$ before the position of maximum intensity.

The only inadequacy of the prediction for conditional intensity i_{ph} on the plume axis is its underestimation of the intensity near the source. Experiments

indicate that inhomogeneous mixing rapidly produces plume fluctuation intensities of $i_p \simeq 1.0$, while the theory predicts a slower rise. Attempts to improve the theoretical prediction near the source by altering the functional form for q_p/Q led to a poorer prediction for the intermittency, and for plume behavior far downwind.

Predicting Plume Intermittency

The only parameter that is still needed to estimate the probability of peak concentrations is the intermittency, γ. With our semi-empirical models for both the total and conditional concentration fluctuation intensities, i, and, i_p, the intermittency can be computed directly without further assumptions. By rearranging (3-6) which gives an exact relation between these three variables the intermittency is

$$\gamma = \frac{i_p^2 + 1}{i^2 + 1} \qquad (3\text{-}14)$$

where (2-18) is used for i and (3-13) for i_p. Thomas (1978) used this same relation to compute γ from experimental measurements of i_p and i in jets and plumes.

Figure 12 shows a comparison of intermittency predicted by inserting the theoretical models for i_p and i in (3-14), with observations in a plume released in a wind tunnel boundary layer. The agreement of theory and experiment is satisfactory, and we see the expected trend of an initially decreasing γ, as large scale turbulence causes meandering of the narrow plume, followed by increasing γ as the plume width increases further downwind. This increasing plume width reduces the proportion of turbulent eddies that are larger than the

plume, and so reduces meandering. In Figure 12 we see clearly that the effect of source size on the total variance is contained almost entirely in its effect on plume intermittency, and not on the process of air entrainment and inhomogeneous mixing within the meandering plume. Figure 11, where intermittency effects are removed indicates little effect of source size on the conditional variance.

Crosswind and Vertical Profiles of Fluctuation Statistics

The spatial distribution of concentration fluctuation statistics i_p and γ is required to predict peak concentrations near the ground, and to integrate across the plume to determine the total risk for changing wind direction. Also, a comparison between measured and predicted profiles of the plume fluctuation intensity and intermittency will provide a good test of the validity of the extended theory for these conditional statistics. The sequence of calculations required to predict i_p and γ are as follows:

1. The total fluctuation intensity i, is computed first, using (2-18) with (2-14) and (2-16) for the variance source strength q/Q. The F and G functions for the vertical and cross-wind distribution are given by (2-2), (2-3), (2-10) and (2-11).
2. The conditional "plume" fluctuation intensity i_p is found by first calculating the plume intensity $i_{p\infty}$ far above the surface by using (3-11) with (3-8) for the plume variance source strength q_p/Q. This result is then inserted in (3-13) to account for the surface turbulence dissipation effects on i_p.
3. The intermittency γ is calculated from the exact relation (3-14) using the previously estimated values for i_p and i.

This same sequence of calculations would be followed in a risk analysis.

The wind tunnel data of Fackrell and Robins (1980a), (1980b) was reanalyzed to generate the conditional "plume" fluctuation statistics from measured values of the total variance, mean and intermittency. These measured values were taken with a fast response (in excess of 300 Hz) flame ionization detector for an elevated source released at about one fifth of the boundary layer thickness above a rough surface. The neutrally stable wind tunnel boundary layer had a scale of about 550:1 relative to the full scale atmosphere. The full scale equivalent release conditions correspond to source height of about 125 meters, and a log-law surface roughness of z_o = 25 cm.

Figures 13 and 14 show a comparison between the theoretical model and measurements at a distance of 0.83 source heights downwind. At this position near the source, the plume had not interacted with the small scale turbulence near the ground, and constant value of conditional "plume" fluctuation intensity i_p was observed in both the cross-wind and vertical profiles. The effect of the ground surface in increasing mean concentration and in dissipating variance is apparent in the vertical profiles shown in Figure 15 and 16 at x/h = 15 and 25 source heights downwind. In Figure 15 we see that the theory tends to underpredict plume fluctuation intensity near the ground, mostly because the Gaussian plume model overpredicts the value of the mean concentration in this region, decreasing the ratio $i_p = \sqrt{\overline{c_p'^2}}/\overline{c}_p$. The tendency of the Gaussian model to over-predict mean concentrations near the ground was discussed in Chapter 2, and is evident in the data shown in Figure 2. The low estimate for plume fluctuation intensity should be viewed not as a deficiency in the theoretical model for total or conditional variances, but rather as an inherent flaw in the

Gaussian mean profile model. This deficiency is balanced by the convenience and simplicity inherent in the Gaussian model.

The effect of the variance sink strength parameter α (which simulates variance dissipation near the surface) is shown in the theoretical profiles of Figures 15 and 16. The data for the elevated source is too scanty near the ground to make a definitive choice for the variance sink strength. The results are consistent with the choice of $\alpha = 0.6$, a value derived from measurements in a ground level source in a wind tunnel boundary layer, (see Figure 6). This choice lies between the extremes of a totally passive surface with $\alpha = 0$, and a strongly dissipating surface with $\alpha = 1.0$, which forces the variance to zero at the surface. It should be noted that very close to the ground there probably is a well mixed layer, with a thickness about 1 to 10 times the log roughness height z_o, where dissipation forces the concentration fluctuations to zero. However, like many other turbulent velocity and concentration profiles, the best fit to data above this layer appears to produce a finite value of variance at the surface, as shown in Figure 6.

One interesting and unexpected result of the theory for the conditional plume statistics is its prediction that both crosswind and vertical intermittency profiles will <u>not</u> produce similarity solutions for a ground level source. This effect is shown in Figure 17 where the theory is compared with ground level source measurements. These intermittency measurements are from the same data set as the variance measurements shown in Figure 6. Although experimental scatter obscures some of the trend, the measurements appear to support the theoretical curves, which show an increasing intermittency factor γ with downwind distance at fixed positions y/δ_y.

Figure 17 also shows that the theory consistently underpredicts the value of the intermittancy factor γ for a ground level source. For the vertical profiles this underprediction appears to be due mostly to the use of the Gaussian approximation for the concentration profile rather than exp $(-A(z/\delta_z)^a)$ with $a < 2.0$. Figure 18 shows the effect of varying the vertical profile exponential function on the predicted intermittency. Using the exponent of $a = 1.7$ of Robins (1978), and Wilson, Robins and Fackrell (1982), agreement between experimental data and theoretical predictions is significantly improved. However, the theoretical model still tends to underpredict intermittancy in the outer edges of cross-wind plume profiles. This underprediction when $y > \sigma_y$ will raise its ugly head when we attempt later to estimate the risk of exposure to peak concentration in the fringes of the plume.

In summary, the agreement between theoretical and measured values is remarkably good considering that the profiles shown in Figures 14, 15 and 16 are not normalized as were the profiles shown in Figures 2 and 3. The only doubt regarding this theory is that it is based on, and validated by, only two sets of data measured in wind tunnel boundary layers. While there appears little doubt that the model is physically sound, further independent measurements are required to determine the effect of atmospheric stability, and to verify the choice of empirical constants for fluctuations very near the surface, where accurate estimates for risk analysis are required. Other data on concentration fluctuation statistics is available in the literature, and in the following section we will review some of these studies.

Other Studies of Concentration Fluctuation Statistics

Most previous work on concentration fluctuation statistics have studied the turbulent jet and its combustion related applications. While there are some

parallels between a jet and a diffusing plume, the essential difference is that jets diffuse as a result of self-generated turbulence whose scales are smaller than the jet dimensions, while a plume often diffuses in large scale eddies which cause plume meandering. Becker, Hottel and Williams (1967) carried out a study of turbulent jet fluctuations, and found that the conditional fluctuation intensity i_p approached a value of 0.73 in the outer fringes of the jet. However, the measurements of Antonia, Prabhu and Stevenson (1975) and Morton and Shaughnessey (1977) suggest that the conditionally sampled fluctuation intensity i_p will not exceed a value of about 0.45 on the plume axis. This is considerably less than the values of 0.9 to 1.3 for elevated plumes in the present study. This essential difference between jets and plumes is reflected in the observations of Becker, Rosenveig and Gwozdoz (1966) who studied concentration fluctuations for a plume released from a point source in the core of turbulent pipe flow. The turbulence scale was sufficiently small compared to source size to produce little plume meandering, and their observations may be interpreted as conditional "plume" variance. On the plume axis their fluctuation intensities i_p ranged from about 0.93 to 1.20, in good agreement with the data reported in the present study, and in marked contrast to the lower intensities observed in turbulent jets. At varying downwind locations they found similarity profiles for plume fluctuation intensity, i_p with a profile shape which supports the present model of a displaced source pair for variance. However, this reassuring agreement is contradicted by the data of Gad-el-Hak and Morton (1979) who used laser doppler techniques to measure both velocity and concentration of smoke in grid turbulence. They observed a concentration fluctuation intensity $i = 0.4$ on the plume axis, decaying slowly downstream. Even considering their large source size, which was

half the diameter of the mesh used to generate the grid turbulence, the theory in the present study predicts a concentration fluctuation intensity about 4 times higher than their observed value.

However, other measurements in full scale atmospheric plumes support the present theory. Jones (1978) carried out short distance diffusion measurements using an ion generator and electron capture detectors with a response time of about 0.1 second. The source was located one meter above ground level and downwind measurements were made a distances of 2 to 15 meters, to determine both intermittency and plume fluctuation intensity. These measurements are summised in Table 3-1. From our point of view, the most significant observation was the almost constant value of plume concentration fluctuation intensity i_p, which had an average of 0.95 for the various downwind positions. While this result supports the wind tunnel observations of Fackrell and Robins (1980a), it again points out the inadequacy of the current theory which predicts a much slower rise of i_p from a value of zero near the source.

Full scale measurements by Eidsvick (1980) in continuous plumes sampled 500 meters from the source gave similar values for fluctuation intensity. Tracer material was released at a height of about 2 meters above the ocean surface in near neutral conditions, and plume traverses were normalized by their centroids to remove the effects of meandering. The conditional fluctuation intensities thus produced gave values of about $i_p \approx 1.35$ on the plume center line and $i_p \approx 1.9$ near the fringes. Eidsvick noted that these values are much larger than the recommended value of Csanady (1973, p. 231) of $i_p = 0.36$ for smooth terrain. Csanady (1973) recommended the following values for various atmospheric conditions.

TABLE 3-1

CONCENTRATION FLUCTUATION IN THE PLUME CLOSE TO A SOURCE NEAR GROUND LEVEL, FROM JONES (1978)

x m	$\lambda = \frac{\sqrt{\sigma_y \sigma_z}}{H}$	ATMOSPHERIC STABILITY			
		NEUTRAL		UNSTABLE	
		i_p	γ	i_p	γ
2	6×10^{-5}	1.07	0.11	1.33	0.09
5	9×10^{-5}	0.81	0.06	0.92	0.04
10	1.4×10^{-4}	0.98	0.08	0.58	0.03
15	1.6×10^{-4}	1.00	0.03	0.08	0.02

Notes: Source height h = 1.0 m

Source diameter d = 0.024 m

	i_p
smooth terrain stable atmosphere	0.20
smooth terrain neutral atmosphere	0.36
rough terrain	0.80

Wilson (1979a), borrowing this idea from Csanady, also suggested varying values for the plume fluctuation intensity near ground level ranging from 0.25 to 1.0 depending on atmospheric conditions and terrain roughness. The present model no longer requires this specification, but rather relies on the scaling of fluctuations in terms of plume dimensions through the parameter λ to account for the effects of stability and roughness, and a variance sink model for ground surface dissipation.

Predictions for the shape of crosswind profiles of total fluctuation intensity by other investigators are shown in Figure 19. The results are in reasonable agreement, and the main difference appears to lie in varying values for the intermittency observed in other experiments. In particular, the data of Netterville (1979) suffers from poor spatial resolution of the fast response probe and resulted in artificially high values of intermittency factor γ, and correspondingly low values of the fluctuation intensity, i. By coincidence, this experimental difficulty caused the data to fall exactly on the line for a single variance source $\phi = 0$. It was only when measurements of Robins and Fackrell (1979) were examined that the present model with a variance source pair was developed, and this line is shown as $\phi = 0.706$ on Figure 19.

Previous Models and Measurements for Intermittency

Most intermittency measurements in turbulent boundary layers have been

correlated using a semi-empirical model developed through the pioneering work of Corrsin and Kistler (1955). This theory modelled the edge of a turbulent boundary layer as a wrinkled sheet that pulsed inwards and outwards at random. They postulated that this boundary had a Gaussian probability distribution for position about some mean location, which resulted in an error function for the vertical variation of intermittency factor γ.

Recent improvements in flow visualization techniques has led to the observation that engulfment by turbulent eddies is an important mode of entrainment making the pulsating sheet model of Corrsin and Kistler less physically realistic. However, the data for concentration intermittency in jets is in good agreement with an error function model for spatial variation, see Becker, Hottel and Williams (1965). With its two adjustable constants of mean boundary position and pulsation standard deviation the error function model also gave a good fit to the data in Figure 17 for intermittency profiles across the plume from a ground level source.

However, for the present model of variance diffusion, the error function model for intermittency has one serious disadvantage. When an error function for γ is used in conjunction with the Gaussian profiles for total mean and variance, $\bar{c}$ and $\overline{c'^2}$, the predicted values of total intensity i_p are no longer constant across the plume. Not only does this disagree with experimental observations (see Figures 13 and 14) but the predicted profiles for i_p are mathematically indeterminate, giving negative values for i_p in the outer fringes of the plume. It was this behavior that led to the adoption of a model which predicts i_p and then computes γ, rather than the reverse.

Summary

In this chapter we have considered the intermittency factor, γ, which describes the fraction of time the plume is present, and the conditional plume statistics measured when plume material is in contact with the receptor. These statistics are essential for estimating the risk of peak concentration probabilities. A simple model which extends the spatial prediction for total variance from a plume in a boundary layer has been developed, and verified using wind tunnel data. Full scale measurements and other experimental studies of plume and jet concentration fluctuations generally support the theory. Now that we are armed with the necessary equations to predict statistics at any point in a plume, we need only determine the form of the probability distribution function to estimate peak concentrations. This will be our next goal.

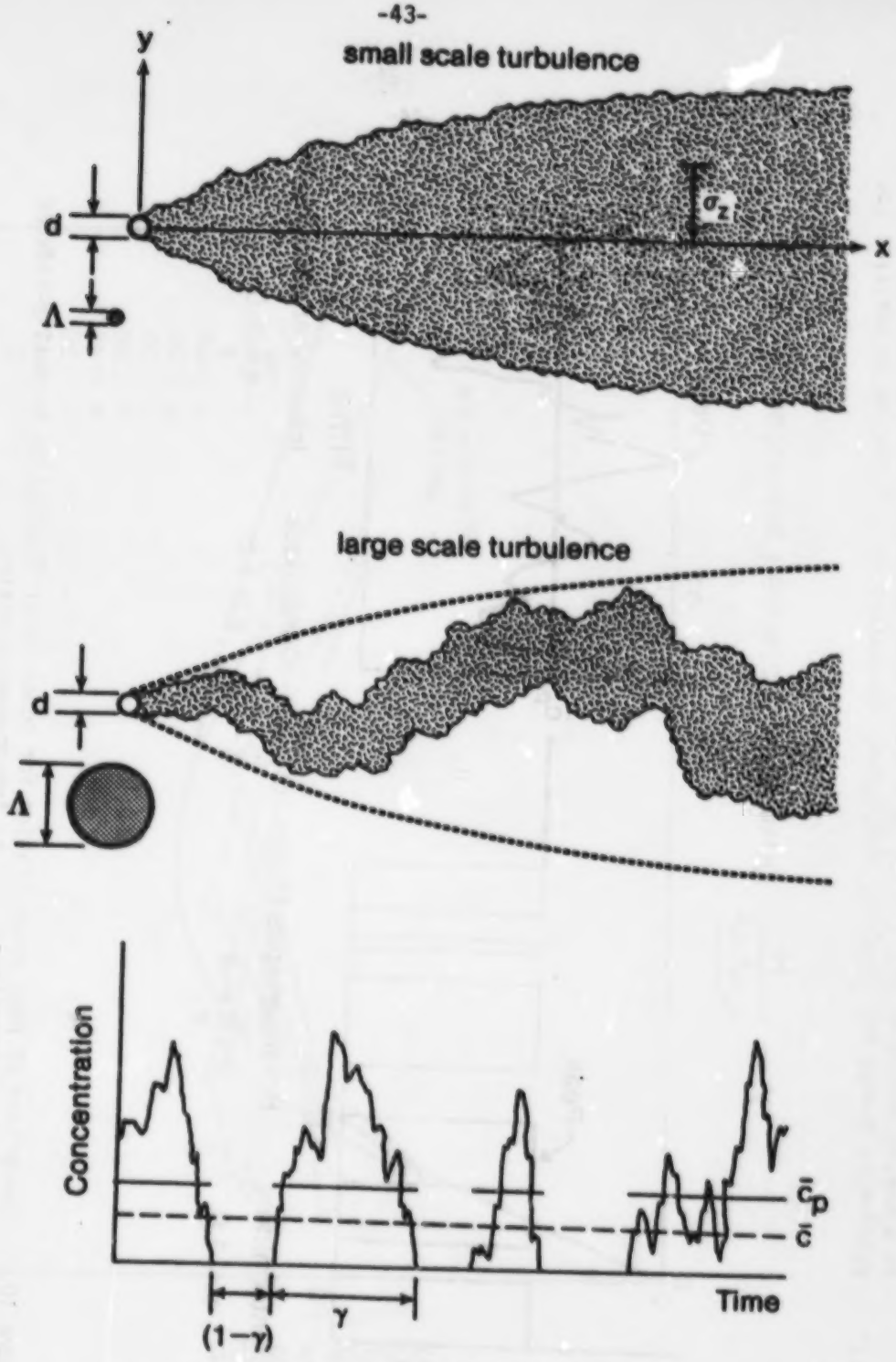

Figure 9. Concentration fluctuations and intermittency caused by plume meandering.

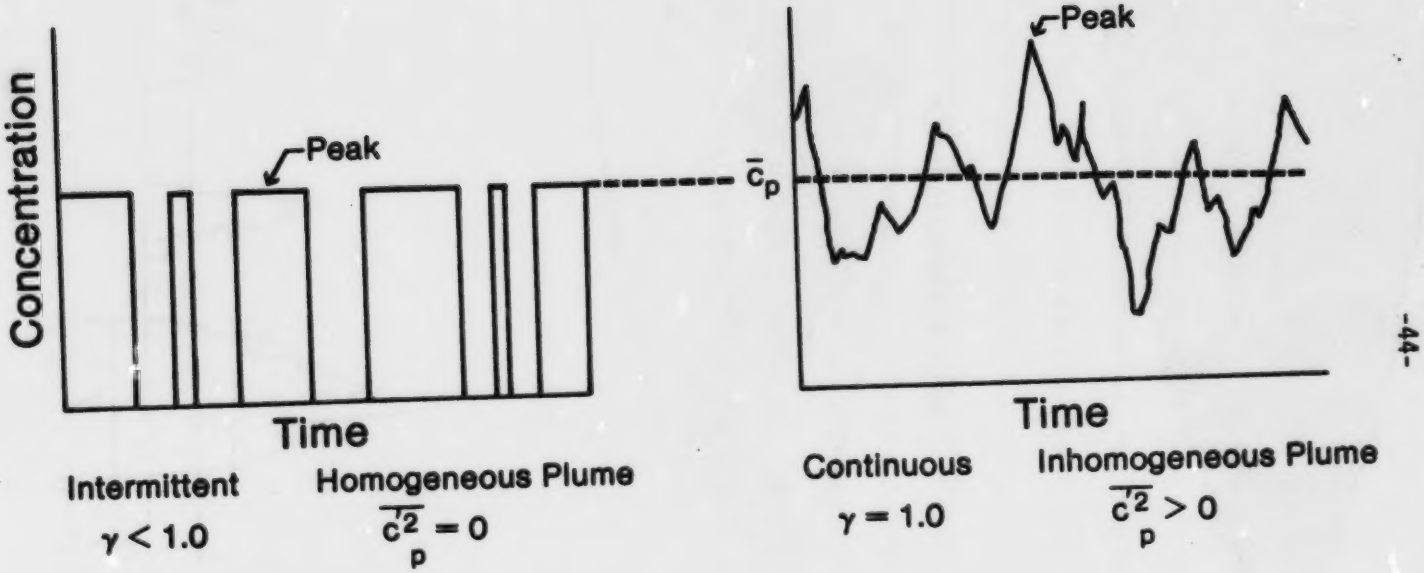

Figure 10. Comparison of peak concentration for equal total fluctuation intensity induced solely by intermittency and by inhomogeneous mixing.

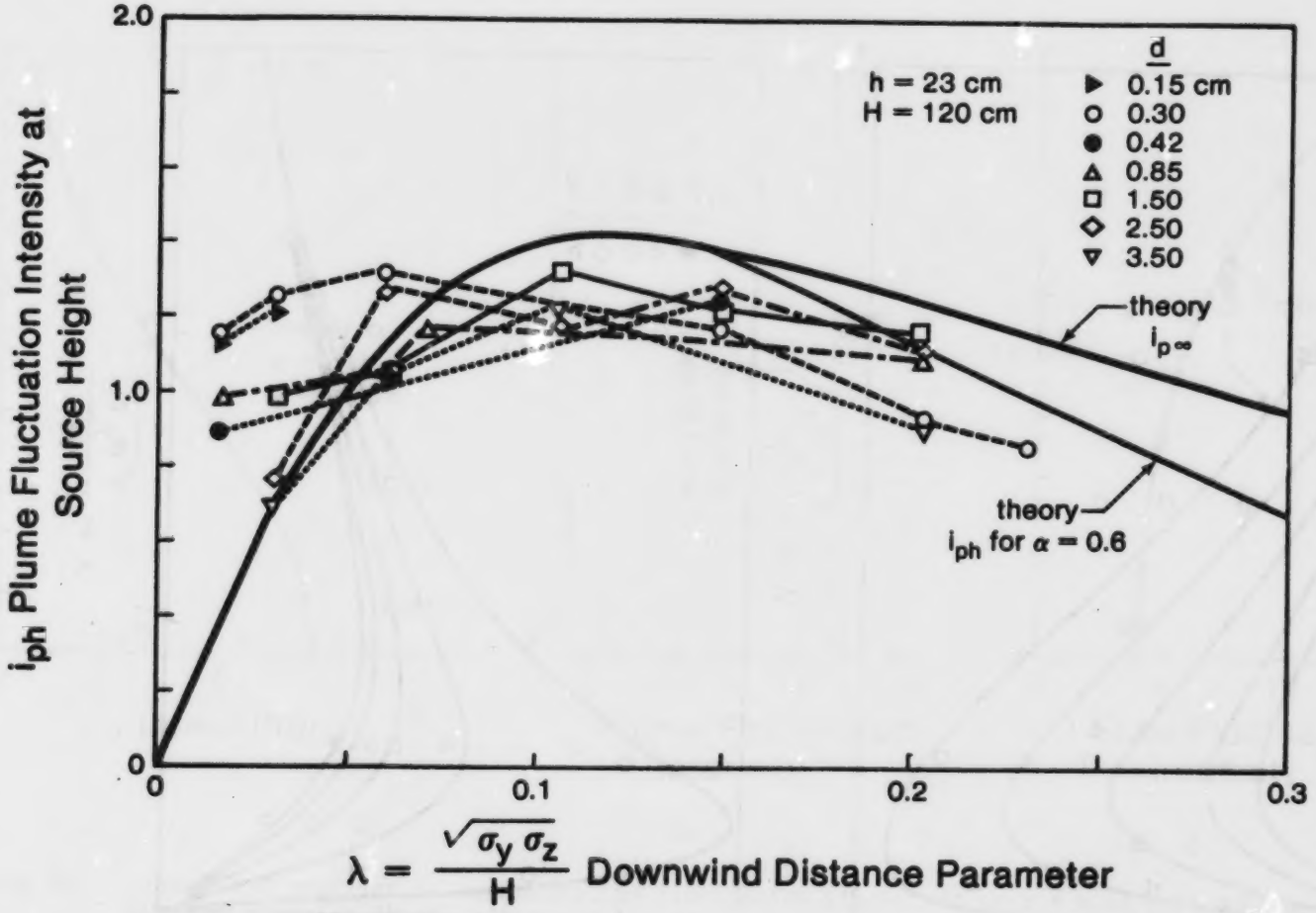

Figure 11. Along-wind growth and decay of conditional concentration fluctuation intensities for an elevated source at h/H = 0.192 in a wind tunnel boundary layer of thickness H. Data from Fackrell and Robins (1980a).

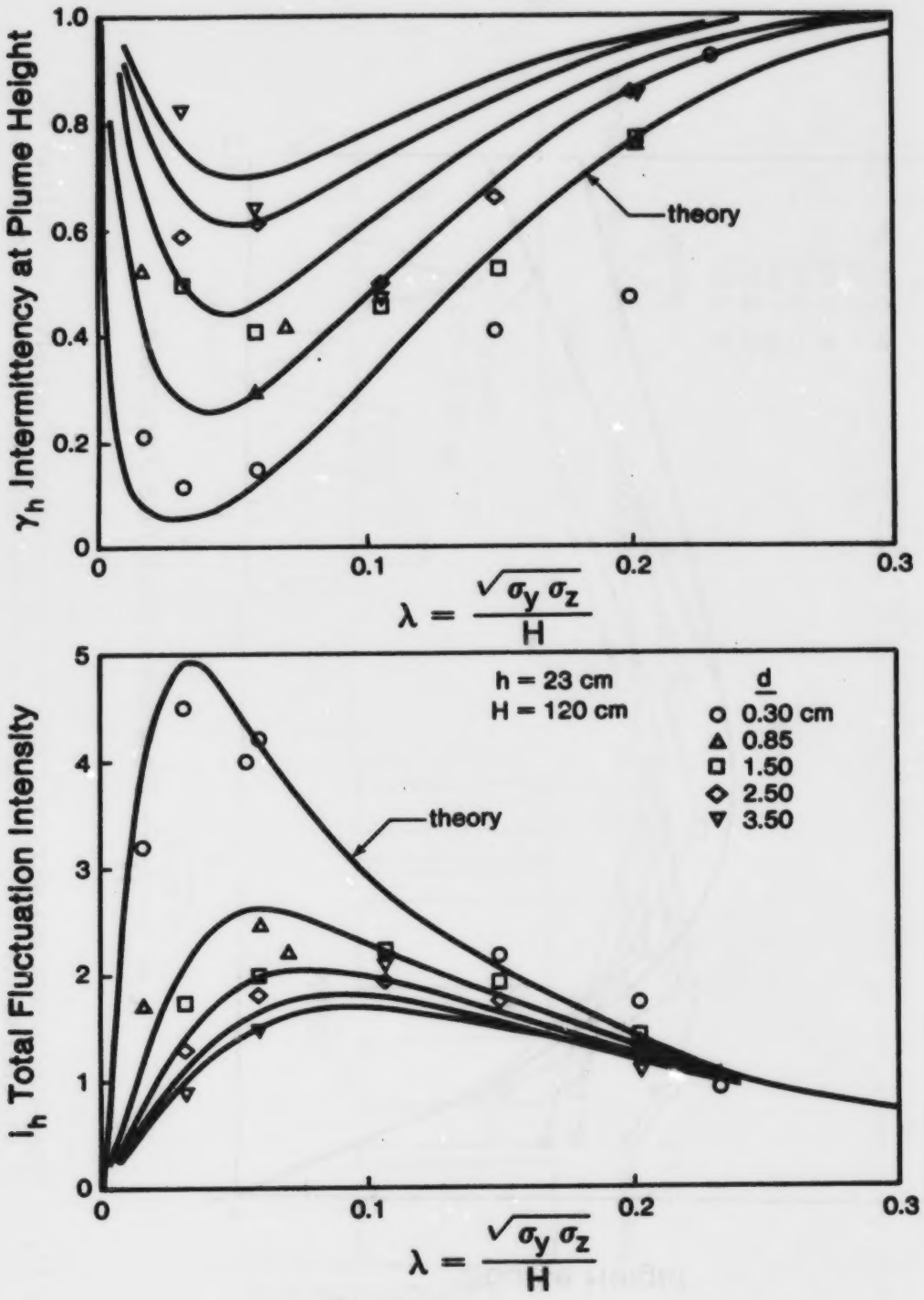

Figure 12. Influence of intermittency on total concentration fluctuation intensity on the plume axis for an elevated source at h/H = 0.192 in a wind tunnel boundary layer of thickness H. Data from Fackrell and Robins (1980a), theory Eqs. (2-18), (3-14).

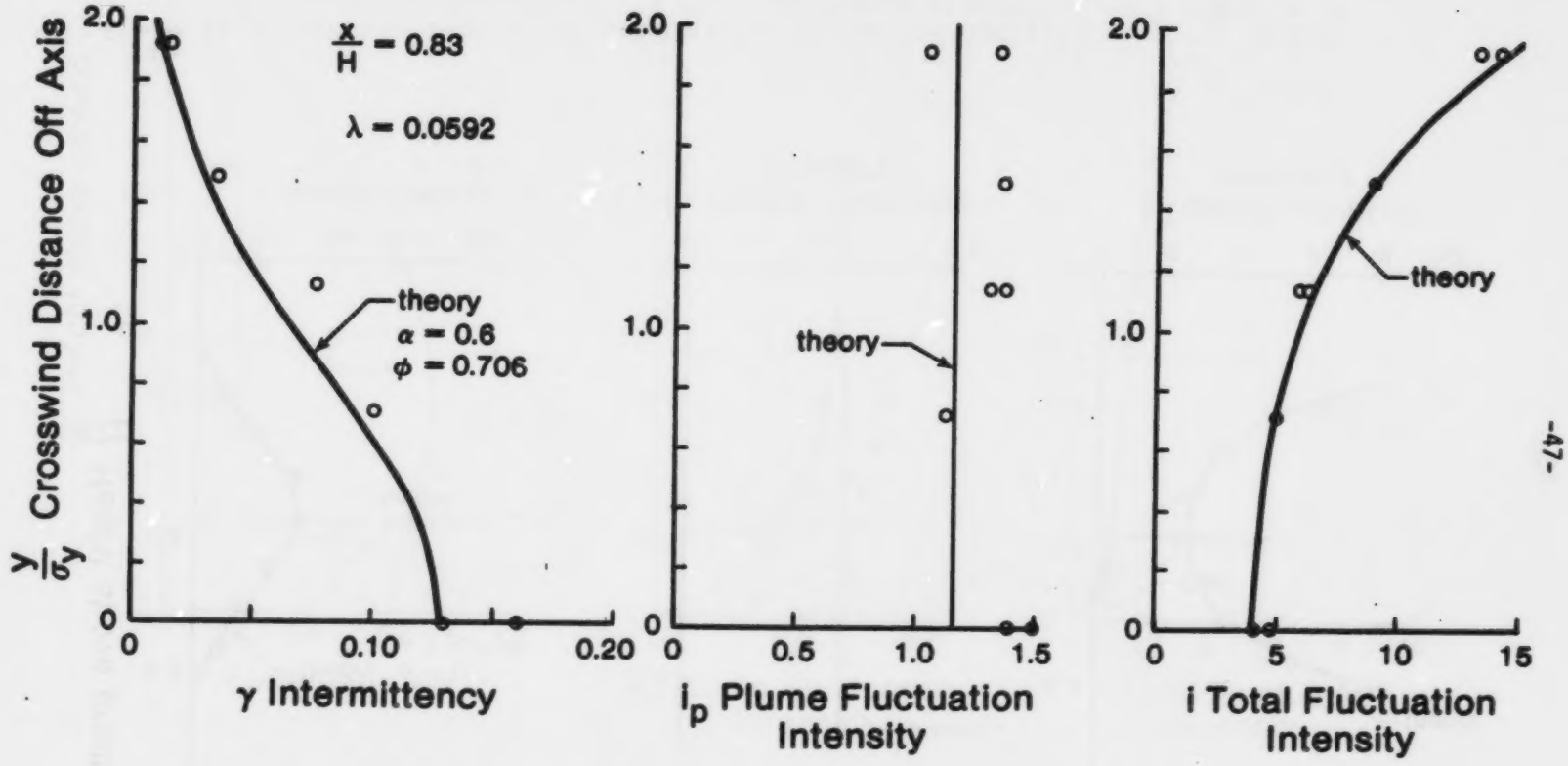

Figure 13. Crosswind profiles of concentration fluctuation statistics on z = h at 0.83 source height downwind in the plume from an elevated source d = 0.85 cm at h/H = 0.192 in a neutrally stable wind tunnel boundary layer of thickness H = 120 cm. Data of Fackrell and Robins (1980a), (1980b).

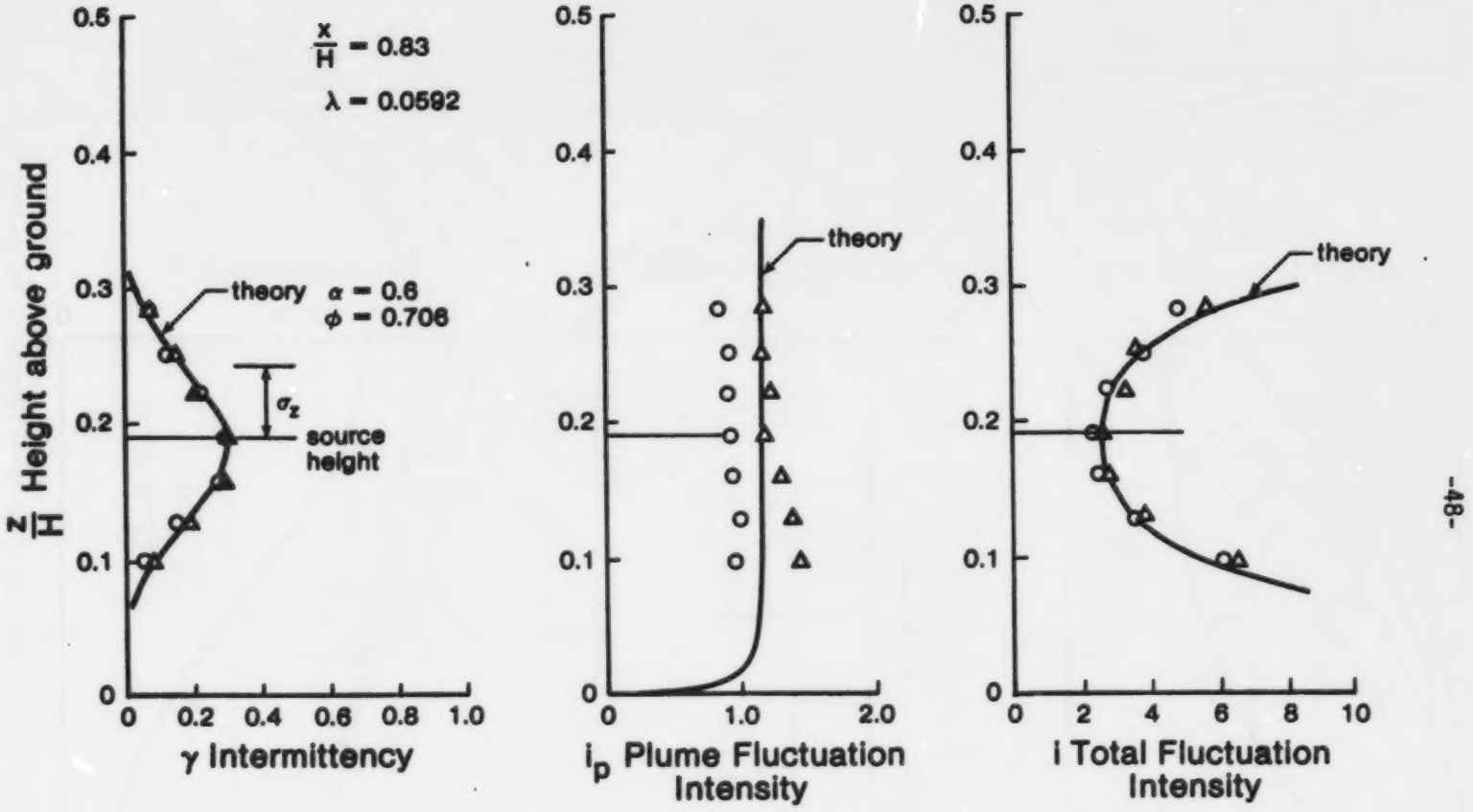

Figure 14. Vertical profiles of concentration fluctuation statistics on y = 0, at 0.83 source heights downwind in the plume from an elevated source d = 0.85 cm at h/H = 0.192 in a neutrally stable wind tunnel boundary layer of thickness H = 120 cm. Data of Fackrell and Robins (1980a), (1980b).

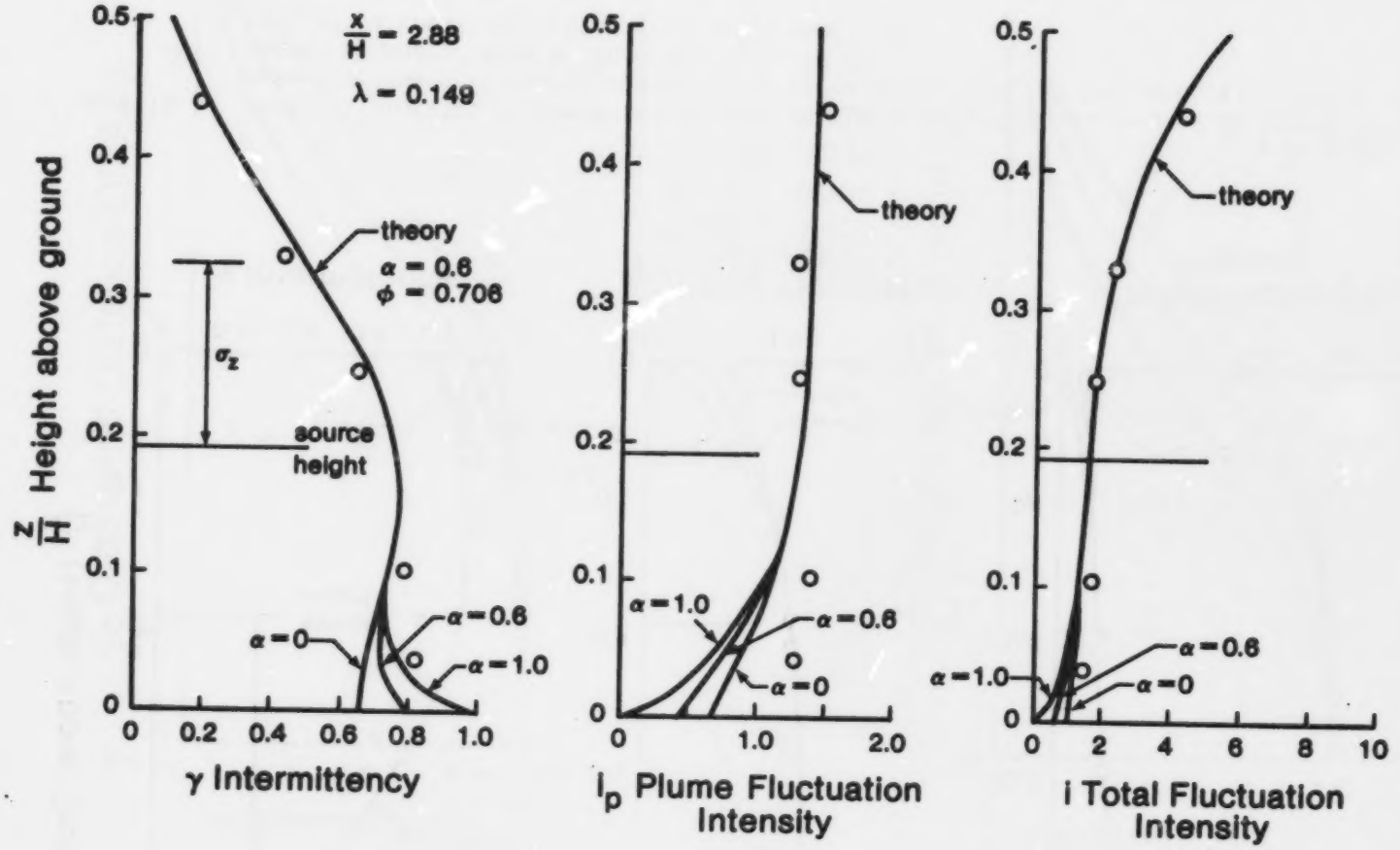

Figure 15. Vertical profiles of concentration fluctuation statistics on y = 0 at 15.0 source heights downwind in the plume from an elevated source d = 0.85 cm at h/H = 0.192 in a neutrally stable wind tunnel boundary layer of thickness H = 120 cm. Data of Fackrell and Robins (1980a), (1980b).

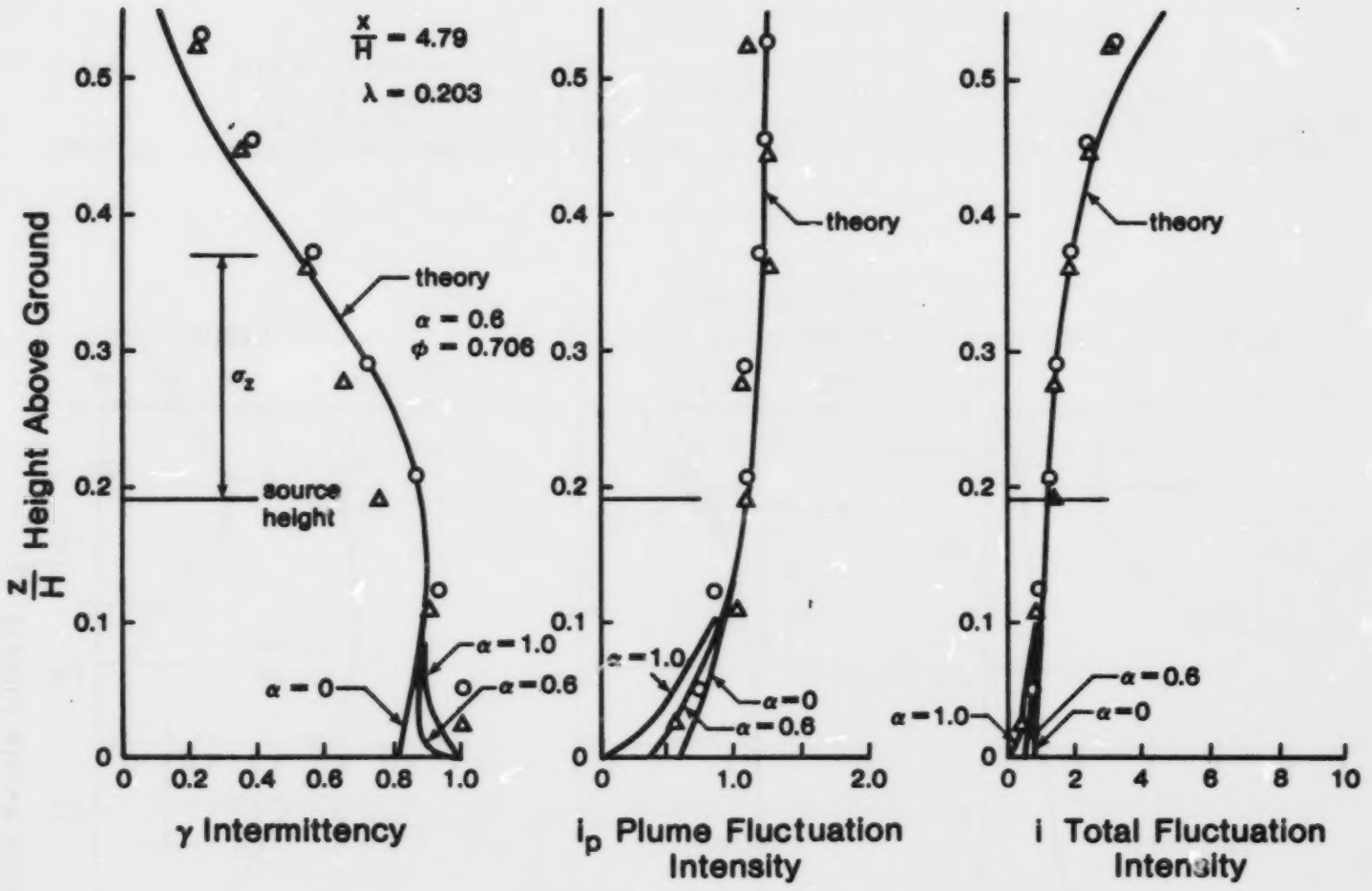

Figure 16. Vertical profiles of concentration fluctuation statistics on y = 0 at 25 source heights downwind from an elevated source d = 0.85 cm at h/H = 0.192 in a neutrally stable wind tunnel boundary layer of thickness H = 120 cm.
Data of Fackrell and Robins (1980a), (1980b).

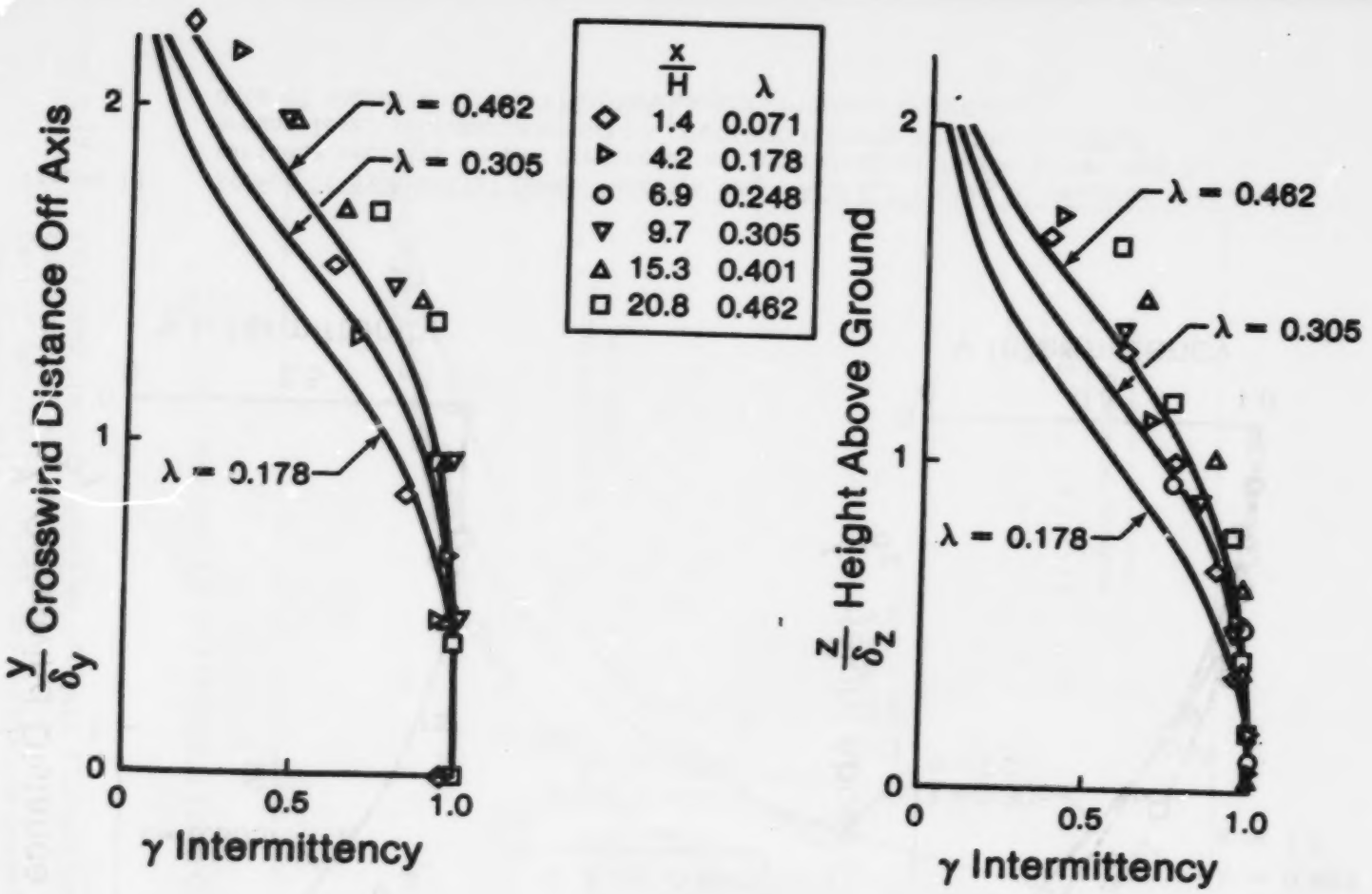

Figure 17. Intermittency profiles in the plume from a ground level source. Data of Robins and Fackrell (1979).

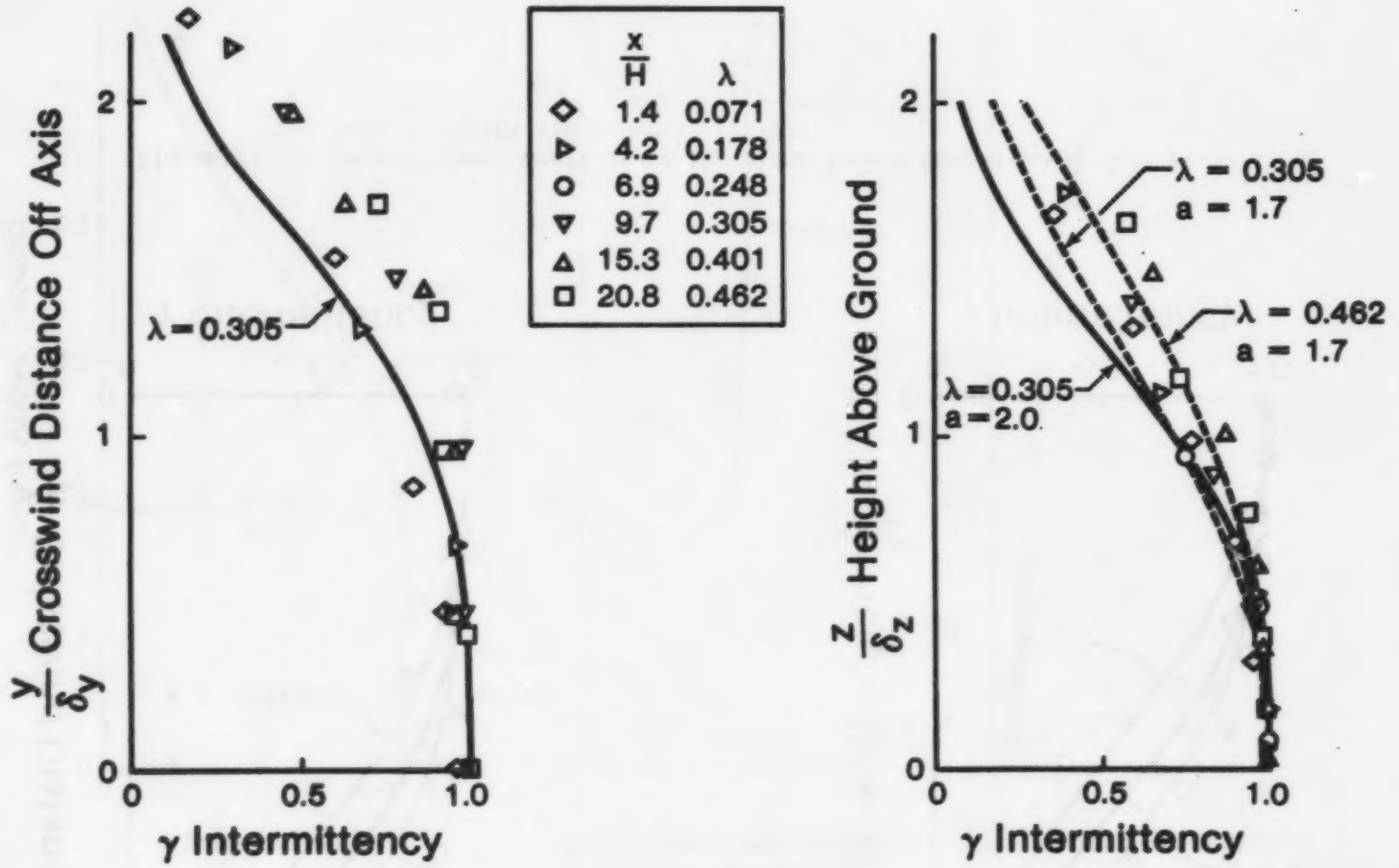

Figure 18. Effect of exponential power function exp $(-A(z/\delta_z)^a)$ used to describe mean and variance profiles on the prediction of intermittency in the plume from a ground level source.
Data of Robins and Fackrell (1979).

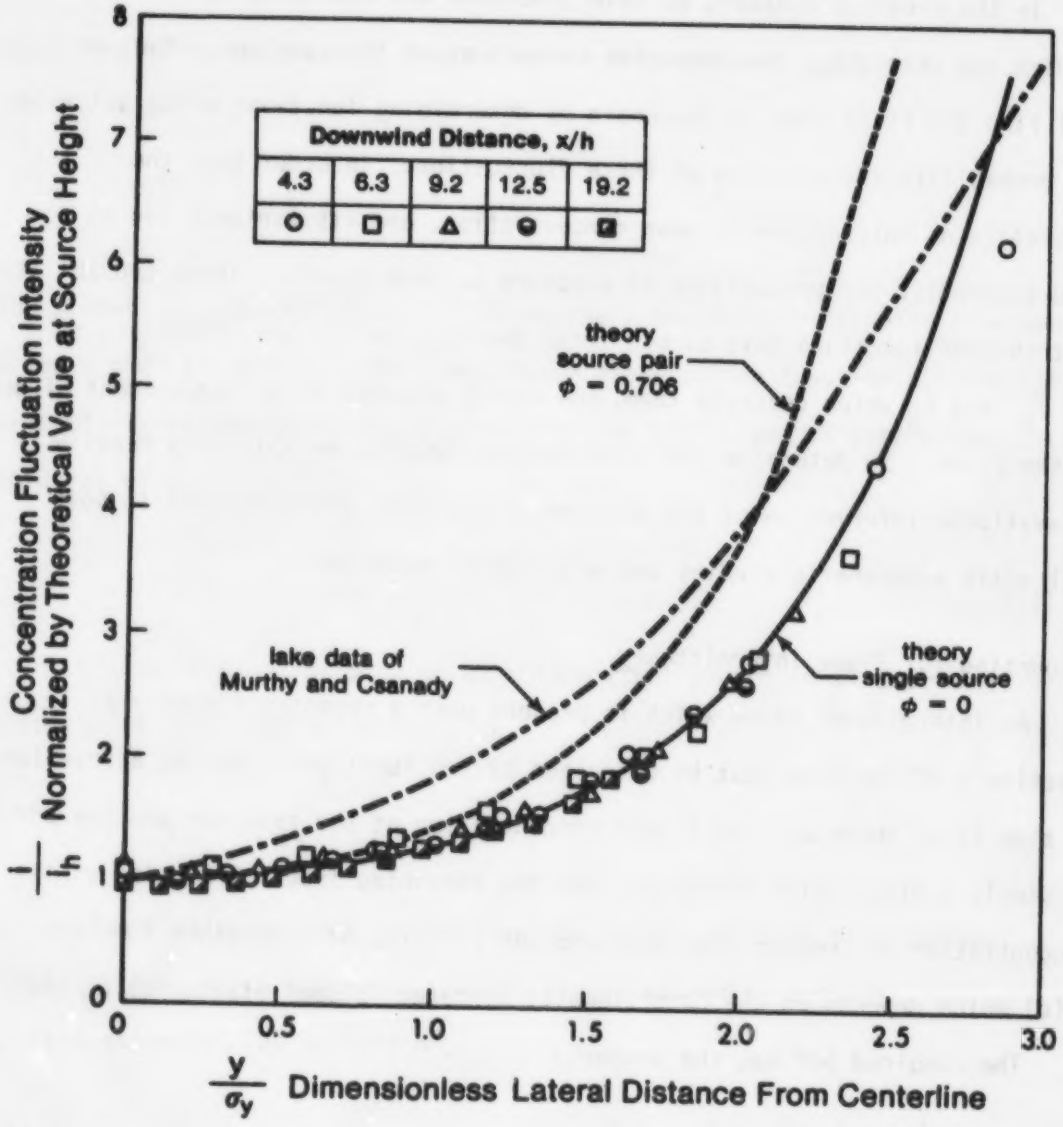

Figure 19. Predicted crosswind profiles of total fluctuation intensity compared with measurements.
Data of Netterville (1979), and Murthy and Csanady, from Csanady (1973 p. 241).

CHAPTER 4

Probability Distribution and Risk of Exposure

In the previous chapters we have developed the necessary equations to predict the statistics that describe concentration fluctuations. Now, we will find the final link in the chain by determining functions which describe the probability distribution of these fluctuations, in order that the statistics of intermittency, mean concentration, and its variance can be used to predict the probability of exposure to peak levels. These probability distribution functions have simple forms because, unlike the statistics γ, and $\overline{c}_p$, and i_p which describe them, the pdf is assumed to be independent of position in the plume. To determine the distribution function we will rely heavily on available information of the measured probability distributions in both full scale atmospheric studies and wind tunnel experiments.

Accounting for Plume Intermittency

An intermittent plume which is present over a receptor for only a fraction γ of the time must be described by two functions. During a fraction of time $(1-\gamma)$ there will be a zero concentration at the receptor and the pdf is simply a Dirac delta function. For the remaining fraction of time γ, the concentration is greater than zero and the pdf will be some other function $p_p(c)$ which depends on the conditionally averaged "plume" statistics $\overline{c}_p$ and i_p. The combined pdf has the property

$$\int_0^\infty p(c)\, dc = 1.0 \qquad (4\text{-}1)$$

The combined pdf is constructed by adding the two individual functions for zero and non-zero concentration

$$p(c) = (1-\gamma)\,\delta(c) + \gamma\; p_p(c) \tag{4-2}$$

Where the delta function is a spike at c = 0 with the properties $\delta(c) = 0$ for $c \neq 0$ and

$$\int_{-\infty}^{\infty} \delta(c) = 1.0 \tag{4-3}$$

The probability distribution p(c) gives the probability of finding c between c and c + dc as p(c)dc. The cummulative distribution function $\Omega(c)$ is the probability of observing a concentration less than c and is simply the integral of p(c)

$$\Omega(c) = \int_0^c p(c)\, dc \tag{4-4}$$

Using the definition for p(c) in an intermittent plume, (4-2) and the property (4-3) of the delta function

$$\Omega(c) = (1-\gamma) + \gamma \int_0^c p_p(c)dc \tag{4-5}$$

For risk analysis the most useful function is the marginal distribution function, MDF, which gives the probability of exceeding a given concentration, c. By definition,

$$MDF = 1 - \Omega(c) \tag{4-6}$$

Choosing a Distribution Function

There are a large number of possibilities for distribution functions to describe concentration fluctuations. However, many functions, such as the Wiebul distribution, are simply sophisticated curve fitting techniques to provide simple functional relationships for existing data. By requiring that the distribution function has some physical basis, the number of candidates are reduced to two, the exponential and log-normal functions.

The exponential distribution function is the simplest of all forms, and has been used successfully by Barry (1977) to correlate full scale concentration fluctuations in intermittent plumes. Meyer (1975) notes that the exponential distribution is a good model for describing random events whose probability of occurrence is small, and individual occurrences have probabilities independent of each other. In terms of concentration fluctuations this implies that an exponential distribution would be increasingly likely as the plume became more intermittent, with short spikes of concentration separated by periods of zero readings. For this case, the probability of observing an individual concentration would be small, and subsequent readings would be uncorrelated because of their separation by periods of zero concentration.

Another possibility is the log-normal distribution. Csanady (1973) notes that a log-normal results when the fractional change in concentration is a random function. If we consider the plume dilution process to be a series of discrete steps, it is logical to assume that at each succeeding step in the dilution process the amount of air entrained is directly proportional to the volume of the eddy at the beginning of the dilution step.

Thus, after the first step, the concentration is related to its initial value c_o by the random dilution factor ξ,

$$c_1 = \xi_1 \cdot c_o \tag{4-7}$$

where $0 < \xi_1 < 1$. After n steps the concentration is

$$c_n = \xi_1 \cdot \xi_2 \text{ -------- } \xi_{n-1} \cdot \xi_n \cdot c_o \tag{4-8}$$

so that concentrations are produced by the <u>product</u> rather than the <u>sum</u> of a series of random variables. Taking the logarithm of both sides

$$\ell n\ c_n = \ell n\ c_o + \ell n\ \xi_1 + \ell n\ \xi_2 + \text{ -------- } \ell n\ \xi_n \tag{4-9}$$

One of the fundamental theorems of statistics states that if a parameter can be described by a sum of independent random variables and n is large, the pdf will be a normal (i.e. Gaussian) distribution. Assuming that the dilution factors ξ_i are statistically independent leads to the result that the logarithm of concentration is normally distributed, or that c follows a log-normal.

Gifford (1972) also argued that a log-normal was likely. He used the experimental observation that concentration is strongly autocorrelated in time so that the time series of concentrations may be thought of as successive dilutions of a single eddy. If, as Gifford claims, strong time autocorrelation is required to generate a log-normal distribution, a non-intermittent plume where concentrations come from a continuous time series of dilutions would be most likely to exhibit log-normal behavior. Thus, from physical arguments we expect that intermittency is the key factor in determining whether a plume displays an exponential or a log-normal form for its probability distribution function.

Harris (1964) carried out a theoretical study which suggests a different physical reason for a log-normal distribution. His interest was not in concentration fluctuations, but in comparing the probability density function of particle travel time from a source to a downwind point on the plume axis. He found that the pdf of particle time travel closely agreed with a log-normal distribution. If the dilution of a puff depends directly on time travel, then the probability density function of the inverse concentration should be approximately log-normal. Bencala and Seinfield (1976) provide the last piece of the puzzle by noting that a variable whose inverse is exactly log-normal will itself closely follow a log-normal distribution. Thus, Harris' studies of particle travel time in turbulent diffusion lead directly to a log normal distribution for concentration fluctuations.

Exponential Probability Density

The exponential function is a very simple form for the conditional "plume" pdf in (4-2)

$$p_p = \frac{1}{\overline{c}_p} \exp\left[\frac{-c}{\overline{c}_p}\right] \qquad (4\text{-}10)$$

This is the continuous form of the Poisson distribution described by Meyer (1975). For mixing processes it is a physically realistic model when the concentrations at a receptor are caused by infrequent eddies separated by long enough time periods that the observed concentration spikes are uncorrelated with each other.

The exponential pdf is expressed in terms of the total mean concentration $\overline{c}$ by using the simple relation of (3-3), $\overline{c} = \gamma\, \overline{c}_p$

$$p_p(c) = \frac{\gamma}{\overline{c}} \exp\left[\frac{-\gamma c}{\overline{c}}\right] \qquad (4\text{-}11)$$

The cumulative distribution Ω_p gives the probability of concentration less than a given c and is found by inserting (4-10) in (4-2) and using this in (4-5) to obtain

$$\Omega_p(c) = (1-\gamma) + \gamma \left[1 - \exp\left(\frac{-c}{\bar{c}_p}\right) \right] \qquad (4\text{-}12)$$

from which the marginal distribution of exceeding c is, from (4-6)

$$\text{MDF} = \gamma \exp\left(\frac{-c}{\bar{c}_p}\right) \qquad (4\text{-}13)$$

The most remarkable property of the exponential pdf is that its conditional variance is identical to the square of the mean, $\overline{c'^2_p} = \bar{c}^2_p$, which results in $i_p \equiv 1.0$ under all conditions. The plume fluctuation intensity is no longer a variable in situations where the exponential distribution applies, and we need only $\bar{c}_p$ and γ to predict risk of peak exposures. Thus, if an exponential distribution applies, all our efforts in Chapters 2 and 3 to develop models for i_p have been a waste of time!

In fact, observed fluctuation intensities in full scale, Barry (1977) and in elevated wind tunnel plumes, see Figure 11, are often near unity except very near the source. However, there appears to be no physical law or constraint that holds $i_p \equiv 1.0$, and an exponential distribution should be viewed as simply an approximation to a complex turbulent mixing process that happens to produce plume intensities near unity.

Near the ground, dissipation of concentration fluctuations by small eddies, and by mean wind shear stretching, produces $i_p \simeq 0.5$, see Figure 16. Clearly an exponential pdf will not apply very close to the ground, not only

because $i_p \neq 1.0$ but also because $\gamma \simeq 1.0$ there, removing highly intermittent, conditions on which the exponential distribution depends.

If this is true, why bother with exponential distributions at all? The answer is that the exponential distribution is a good approximation to experimental observations, notably those of Barry (1977), and also seems to be a reasonable model in highly intermittent plumes.

Log-Normal Probability Density

The log-normal distribution is an appealing choice because it has both a physical justification and uses all three statistical parameters $\bar{c}_p$, γ and i_p. The conditional pdf for finding concentration between c and c + dc is

$$p_p(c) = \frac{1}{\sqrt{2\pi}\,\sigma_{\ell p} c} \exp\left[\frac{-\ell n^2\left(\frac{c}{c_{mp}}\right)}{2\sigma_{\ell p}^2}\right] \qquad (4\text{-}14)$$

The parameters $\sigma_{\ell p}$ and c_{mp} are the conditional "plume" log standard deviation and the conditional median concentration, which are related to the measured plume statistics by.

$$c_{mp} = \frac{\bar{c}_p}{\sqrt{1 + i_p^2}} \qquad (4\text{-}15)$$

$$\sigma_{\ell p} = \sqrt{\ell n\,(1 + i_p^2)} \qquad (4\text{-}16)$$

Using (3-3) and (3-6) these may be expressed directly in terms of the measured total mean, $\bar{c}$, and intensity, i, as

$$c_{mp} = \frac{\bar{c}}{\gamma\sqrt{\gamma\,(1 + i^2)}} \qquad (4\text{-}17)$$

$$\sigma_{\ell p} = \sqrt{\ell n\,(\gamma(1 + i^2))} \qquad (4\text{-}18)$$

The cumulative distribution function for the probability of finding concentration less than some specified level, c, is found by combining (4-2) and (4-14) and then integrating, using (4-5) to get

$$\Omega(c) = (1 - \gamma) + \frac{\gamma}{2}\left[1 + \text{erf}\left[\frac{\ell n\left(\frac{c}{c_{mp}}\right)}{\sqrt{2}\ \sigma_{\ell p}}\right]\right] \qquad (4\text{-}19)$$

which is in terms of the error function integral. The marginal distribution function (MDF) for the probability of exceeding c is simply (1-Ω), or

$$\text{MDF} = \gamma\left[1 - \frac{1}{2}\left[1 + \text{erf}\left[\frac{\ell n\left(\frac{c}{c_{mp}}\right)}{\sqrt{2}\ \sigma_{\ell p}}\right]\right]\right] \qquad (4\text{-}20)$$

Observed Probability Densities

The probability density is a much more sensitive means of testing various candidate functions than is the cumulative distribution, which is the area under the probability density curve. Unfortunately, the probability density requires long data records with stable mean values, while the cumulative distribution can often be accurately constructed from data records that are a fraction as long as those required for the probability density. Because of this, virtually all full scale atmospheric data is used to generate cumulative distributions, and only wind tunnel data is available for probability densities.

The distinctly different shapes of the pdf's for log-normal and exponential distributions are shown schematically in Figure 20. The exponential function is physically realistic only for highly intermittent plumes which have a conditional fluctuation intensity of unity. While a log-normal has the capability of dealing with fluctuation intensities different from unity, it has the physically unrealistic property that the probability of observing a concentration only slightly greater than zero, is very small, while at the same time there is a large probability of observing zero concentration, when intermittency removes the plume from the receptor. In highly intermittent plumes where there is a significant probability of zero concentration, the exponential model realistically predicts a smooth monotonically increasing probability of observing small concentrations.

The "three parameter" log-normal of Ott and Mage (1976) provides a more realistic transition between a purely log-normal distribution, applicable to non-intermittent plumes, and an exponential for highly intermittent conditions. In their model the concentration is simply displaced by an offset factor which produces an area under the log-normal curve to the left of the origin equal to the intermittent time period $(1-\gamma)$. This empirical curve shifting technique is shown in figure 20, where it is apparent that increasing plume intermittency, which results in smaller values of γ, will result in the curve being displaced far to the left and having its peak clipped off. This will generate a smooth transition between a log-normal shape and a shape which closely resembles an exponential. Fackrell's (1978) data on plume concentration statistics in a wind tunnel simulation of a stack plume gave excellent agreement with this three parameter log-normal for intermittent concentrations at a ground level receptor. However, both

Fackrell, and Ott and Mage did not directly apply measured intermittency values to determine the necessary offset shift, but rather adjusted the offset to obtain the best fit to the probability distributions. With this extra degree of freedom there is little wonder that their empirical model gave better fits than an intermittent log-normal could have. Because the three parameter log-normal is mostly an exercise in curve fitting, with only a tenuous physical basis, it will be abandoned in favor of simpler models.

Figure 21 shows a comparison between a log-normal distribution and wind tunnel measurements for a ground level receptor under an elevated wind tunnel plume. The slow time response of the wind tunnel concentration detector (about 100 seconds full scale equivalent) was largely responsble for the observed intermittency factor γ=1.0. While the log=normal distribution is not a very good representation of the probability density for low concentrations, it was a good approximation for peaks that exceeded about 1.5 times the mean value. For a variety of receptor locations and plume release heights a log-normal approximation predicted the peak concentrations exceeded 5% and 10% of the time with an accuracy better than $\pm$ 10%. This result was confirmed by the data by Netterville (1979) shown in Figure 22. His measurements using a fast response detector in a small wind tunnel also showed that a log-normal gave good predictions for the probability density function at high concentrations above 1.5 times the conditional "plume" mean value. This data also shows the problems caused by electronic noise in attempting to measure probability distributions. Here we see that random electronic noise with a variance only 0.4% of the concentration fluctuation variance causes the probability density to be smeared out, indicating negative

concentration values, and completely obscuring the density function trend near zero concentration. For risk analysis this is not a serious deficiency because our main interest lies in predicting peak concentrations.

Gad-el-Hak and Morton (1979) used laser techniques to measure concentration fluctuations in a smoke plume released in wind tunnel grid turbulence. In the highly intermittent outer fringes of the plume they observed a monotonically decreasing pdf, while along the axis of the plume where intermittency was near unity, they found a skewed distribution. While no specific functions were fitted to their data, the results support the physical arguments for the conditions under which exponential and log-normal distributions occur.

Observed Cumulative Distributions

Because cumulative distributions require less data to define their shape than do probability density functions, most full scale observations are presented in this way. The majority of these full scale air quality measurements are correlated using a log-normal cumulative distribution. In spite of the fact that these measurements are for long data records, often up to a year in length, it is tempting to use these measurements as a justification for a log-normal model for short term plume fluctuations. Unfortunately, the log-normality of long term urban concentration statistics follows a log-normal distribution for entirely different reasons than do short term fluctuations.

The clue to explaining the log-normal behavior of long term urban air quality statistics lies in the observations of Hanna (1978) who found a strong correlation between urban air quality and wind speed in the form

$$C \propto \frac{Q}{AU} \qquad (4\text{-}21)$$

where $\bar{c}$ is the short time mean concentration, and Q is the total rate of release from a uniform area source that covers the area A. Knox and Lang (1974) also pointed out that for point sources the concentration at a receptor is inversely related to wind speed by (2-1) which predicts $\bar{c} \propto Q/U$. They then went on to conclude that the pdf of urban concentration for long time periods of days to months should be the same as the pdf of U^{-1}, the inverse of windspeed. Bencala and Seinfield (1976) went further, to show numerically that the observed log-normal distribution for windspeed U also leads to an almost exactly log-normal distribution for c. Thus, we see that it is the log-normal behavior of windspeed U, or inverse windspeed U^{-1}, that leads to cumulative log-normals for long term urban air quality concentrations. In an interesting post-script to these urban air quality measurements, de Nevers, Lee and Frank (1979) found that the sudden changes in slope of the long-term cumulative probability distribution for total suspended particulate material at several locations could be explained as random sampling problems for small numbers of observations from a truly log-normal distribution. In summary, while log-normal distributions are receiving ever increasing support for long term concentration fluctuations, they do little to aid us in selecting an appropriate distribution function for the short term fluctuation caused by inhomogeneous mixing in pipeline rupture plumes.

Jones (1978) for short distance fluctuation measurements up to 15 meters from a point source near the ground found that a log-normal cumulative distribution was a reasonable approximation to rapid fluctuations with time periods from 0.1 to several seconds.

Hinds (1969) released a flourescent pigment (zinc sulfide) as a tracer from a point source 1.0 meters above ground, and collected samples on an array of detectors downwind. The concentration fluctuations were derived from 5 second averaging times, and thus represent short term inhomogeneous mixing fluctuation statistics of the type desired for pipeline rupture risk analysis. Figure 23 shows the results of four tests, two with a release immediately upwind from a building, and sampling in its wake.

A log-normal distribution using directly measured values of fluctuation intensity and intermittency are compared with straight line exponential cumulative functions fitted by Barry (1977). For three of the tests where the fluctuation intensity is near unity, there is little to choose between the fit of log-normal and exponential functions. However, for test W6 at the edge of a building wake, where the fluctuation intensity was 2.48, the log-normal is clearly superior in estimating the marginal probability distribution.

Close agreement between exponential and log-normal distribution functions in predicting the probability of exceeding some given peak concentration is also apparent in Figure 24. This figure shows the data of Barry (1977), who correlated full scale measurements several kilometers downwind from a stack plume seeded with radioactive tracer. For his fast response radiation detector, the measurements were highly intermittent, with short time averages indicating that the plume was present less than 10% of the time. Part of this high intermittency was caused by the long data records that included diurnal shifts in wind direction. For both one hour and a 24 hour data averaging times the log-normal distribution was computed using the measured intermittency

factor γ and an assumed conditional fluctuation intensity, i_p, of unity, to allow direct comparison with the exponential distribution which has $i_p = 1.0$. The close agreement between log-normal and exponential marginal distributions depends entirely on this choice for a fluctuation intensity of unity. For other fluctuation intensities the distribution functions rapidly diverge.

In summary, the full scale measurements of cumulative distributions lead us to the following conclusions.

1. The apparent log-normal behavior of long term fluctuation statistics is due to a log-normal distribution for the average wind speeds, and sheds no light on the short term fluctuation probability distribution.

2. Both exponential and log-normal distributions give good agreement in predicting fluctuations for averaging times from 5 seconds to 24 hours. This is due however to the near unity value for conditional concentration fluctuation intensity in many practical situations.

3. For conditions when the concentration fluctuation intensity is different from unity, the log-normal distribution which accounts for varying i_p is clearly superior.

With these observations in mind, it is recommended that a log-normal probability distribution with intermittency be used to predict pipeline rupture peak concentrations. However, further measurements are required before small concentrations between zero and the conditional mean $\overline{c}_p$ can be confidently estimated. Fortunately, these small concentrations are of little importance in a risk analysis.

Core-Bulk Structure in Plume Mixing

Several experiments on turbulent jet mixing with combustion applications have led to the conclusion that there may be a dual structure in the turbulent mixing process. Eddies which arrive at the receptor carrying source material

appear to have one of two entirely different probability distribution functions which superpose to produce the observed pdf. This dual structure is apparent in the concentration fluctuation probability densities measured by Birch et al (1978), and shown in Figure 25. These pdf's were measured at varying crosswind distances from the axis of a methane jet in air. We see what appears to be a highly intermittent structure with an exponential distribution superposed on non-intermittent mixing with a skewed distribution that might be log-normal. Similar observations of this double-humped pdf have been made by Krollman (1981), also in turbulent jets.

Several questions come to mind:

1. What causes material emitted from the source to follow two apparently different time histories?

2. Can a theoretical model be formulated which accounts for this dual structure?

3. If such dual structure fluctuations have only been observed near the orifice of a turbulent jet, is it really necessary to include them in a plume risk analysis model?

To answer these questions we turn to the theoretical work of Chatwin and Sullivan (1979) who in this and earlier papers outlined the basis for a dual structure for the turbulent mixing process. They envisioned the mixing of a puff of contaminant into the ambient air as resembling a loosely tangled ball of wool in which high concentration filaments of source material were gradually distorted and diffused into pockets of low concentration entrained ambient air and previously diffused source material. They referred to the high concentration filaments as the "core", and the low concentration material into which it was continually mixing as the "bulk". While Chatwin and Sullivan offer no suggestions to the forms for probability distribution functions in the core and bulk structure, we can extend their hypothesis

by considering the dilution process as a parallel series of discrete steps.

We envision the dilution of the "core" as a series of small steps in which high concentration eddies are entrained by other eddies with high concentration. The process has no periods of intermittent zero concentration values, and should follow a log-normal distribution according to our previous arguments. The bulk structure, on the other hand, is envisioned as those eddies in the fringes of the plume which are engulfing or are engulfed by, eddies of ambient air with zero concentration. In this low concentration mixture the observation of occasional bursts of non-zero concentration are relative rare events, separated far enough in time to be uncorrelated, so that an exponential pdf is appropriate.

With these rather simple, and hopefully plausible assumptions, the only variable that must be specified is the fraction of time ϵ, that the core structure is present out of the total fraction of time γ that concentrations greater than zero are being observed. Once the core fraction ϵ is known the total probability density function may be computed from an extension of (4-2)

$$p(c) = (1-\gamma)\,\delta(c) + \gamma\,[\epsilon\,p_c(c) + (1-\epsilon)\,p_b(c)] \qquad (4\text{-}22)$$

where p_c is the log-normal "core" pdf given by (4-14) and p_b is the "bulk" pdf given by the exponential (4-10). At this point our attempt to theoretically model a dual mixing structure is thwarted by a lack of information on the conditional mean concentration and intensity in the core and bulk, as well as the problem of estimating the core fraction ϵ. To facilitate an order of magnitude estimate the following rather arbitrary simplifying assumptions are made:

1. The core structure is present for the same fraction of time as the entire non-zero contractions are present. That is, $\epsilon = \gamma$. In the asymptotic limit as the plume becomes more intermittent, with $\gamma \to 0$, the core structure will vanish and the mixing will be governed by an exponential pdf, as expected.
2. The plume intensity in the "bulk" structure must be $i_p = 1.0$ to accommodate an exponential pdf. Assume then that all changes in i_p occur in the "core".
3. The conditional mean concentration $\overline{c}_p$ is the same in the core and bulk structure.

These assumptions were used to compute the combined exponential, log-normal pdf's shown in Figure 26.

As expected, the major influence of the dual mixing structure is on probabilities at concentrations less than $\overline{c}_p$ (which is $c/\overline{c} \leq 2.0$ for $\gamma = 0.5$ on Figure 26). For peak concentrations with $c > \overline{c}_p$ the dual mixing structure has very little effect on the marginal probability of exceeding some fixed peak level.

Thus, while a core-bulk structure may be of crucial importance in determining such important mixing phenomena as the probability of ignition of a combustible mixture it appears to have little effect on the high concentration peaks important in toxic gas exposure risks. We conclude, with a certain amount of relief, that this complicated core-bulk model is not required for risk analysis.

Giving thanks for such small mercies, we are now equipped with simple functional models for γ, $\overline{c}_p$ and i_p, which along with our intermittent log-normal distribution can finally be used to compute risk of exposure.

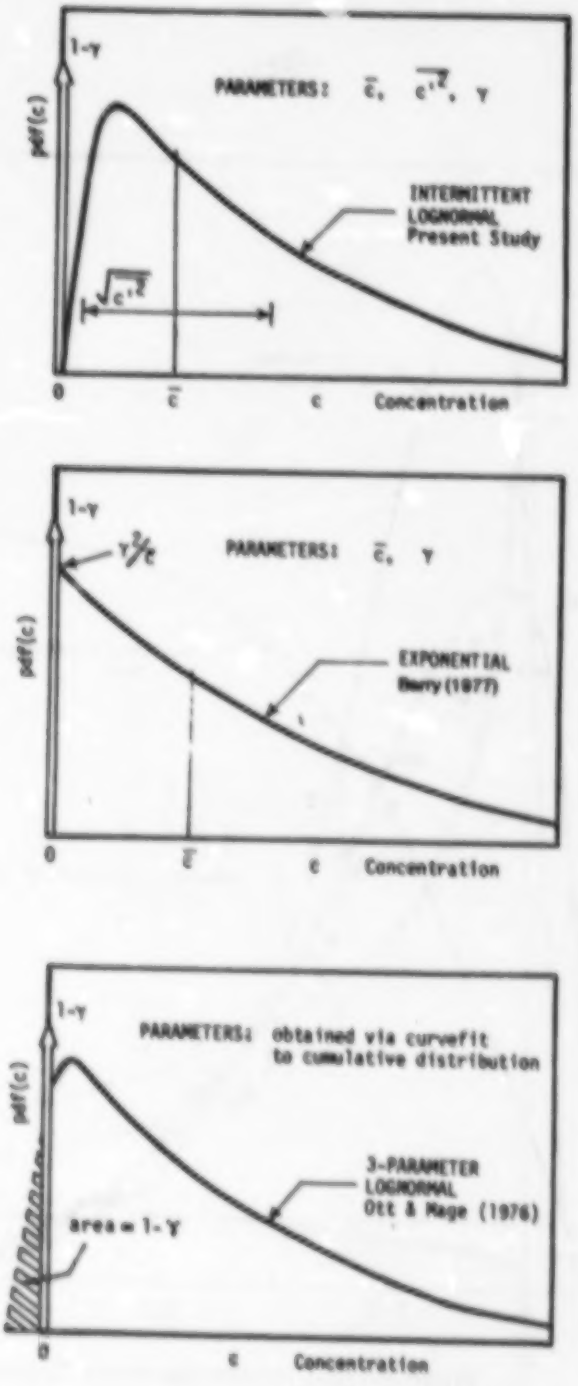

Figure 20. Log-normal and exponential functions for the probability distribution of concentration fluctuations.

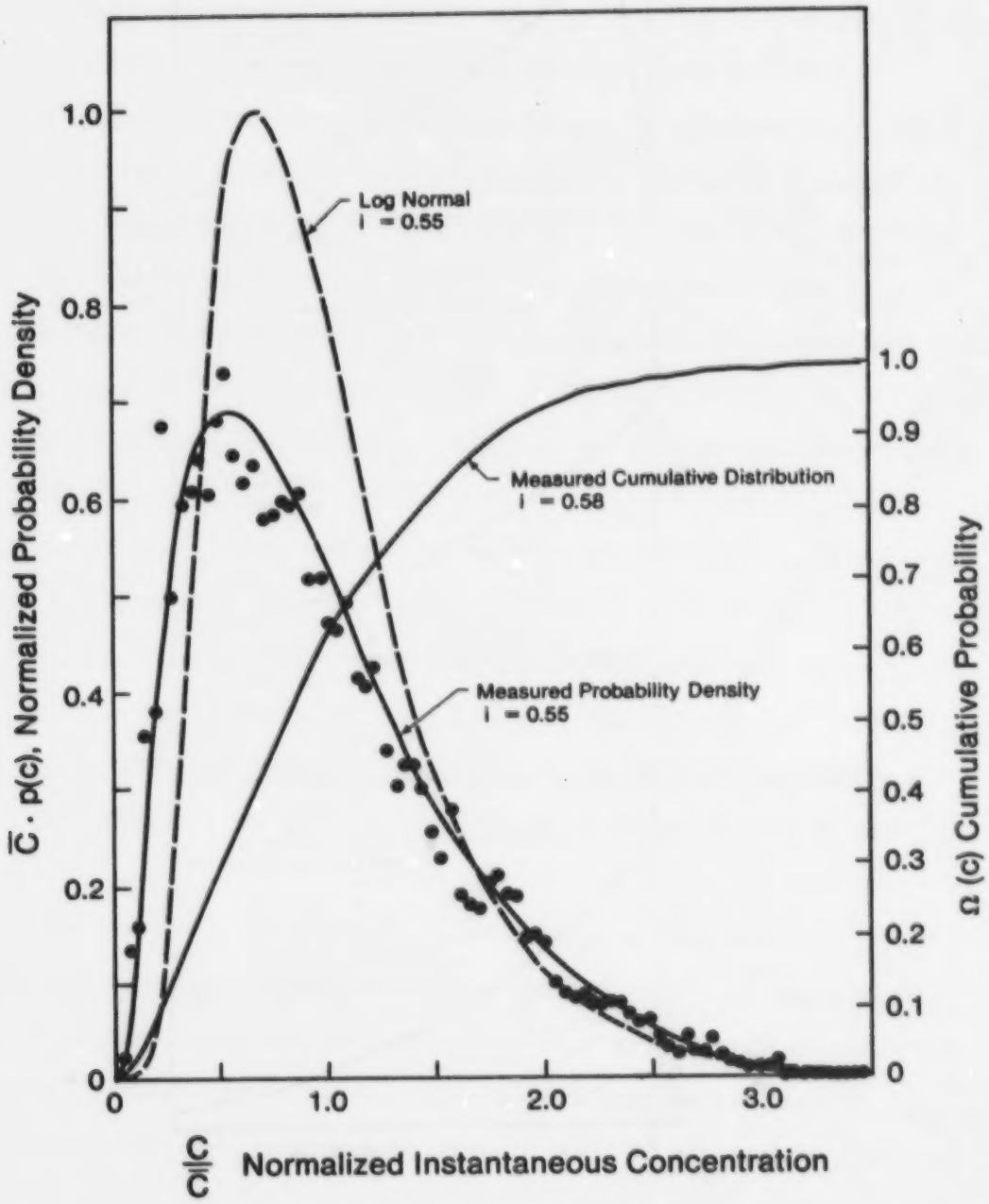

Figure 21. Log-normal probability compared with ground level measurements in a wind tunnel boundary layer.
h/H = 0.44, h/H = 3.2, λ = 0.175,
From Wilson (1979b).

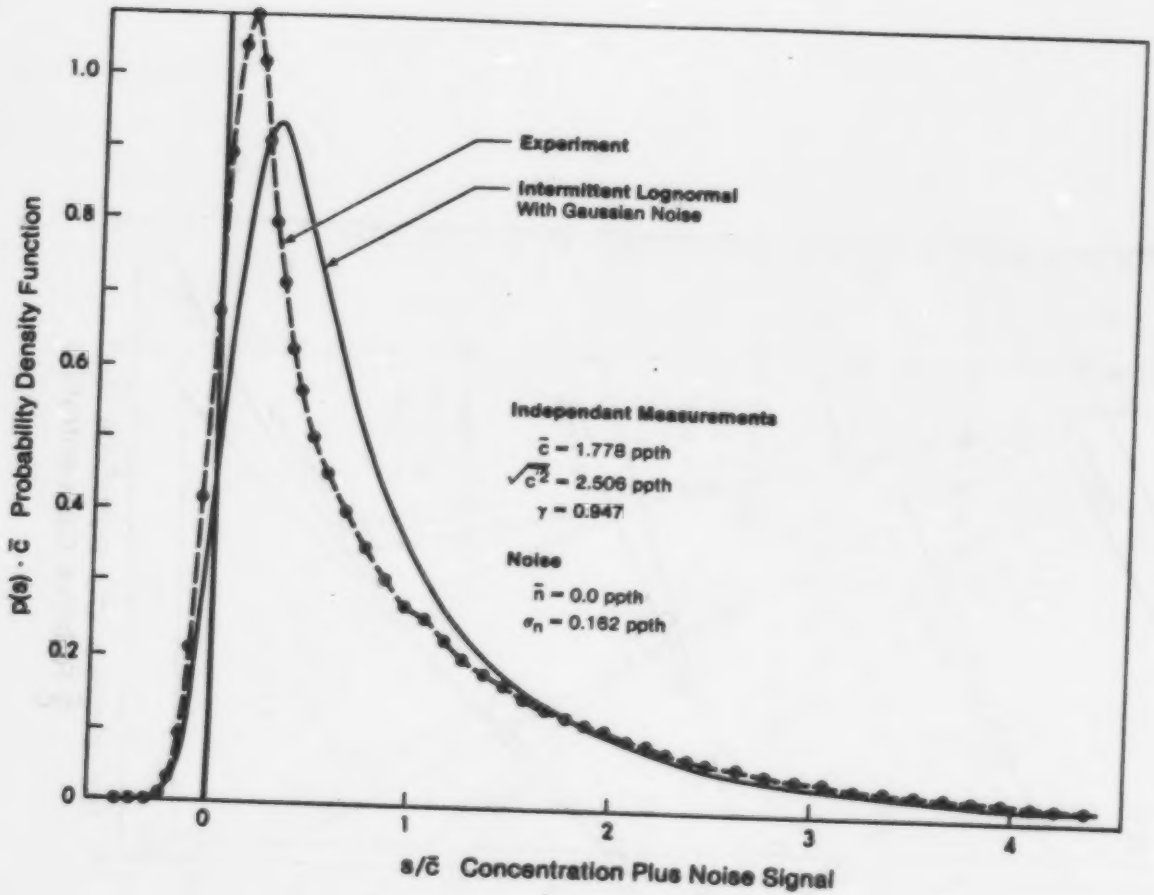

Figure 22. Log-normal probability distribution compared with measurements in a wind tunnel plume at x/h = 12.5, z/h = 0.5 from Netterville (1979).

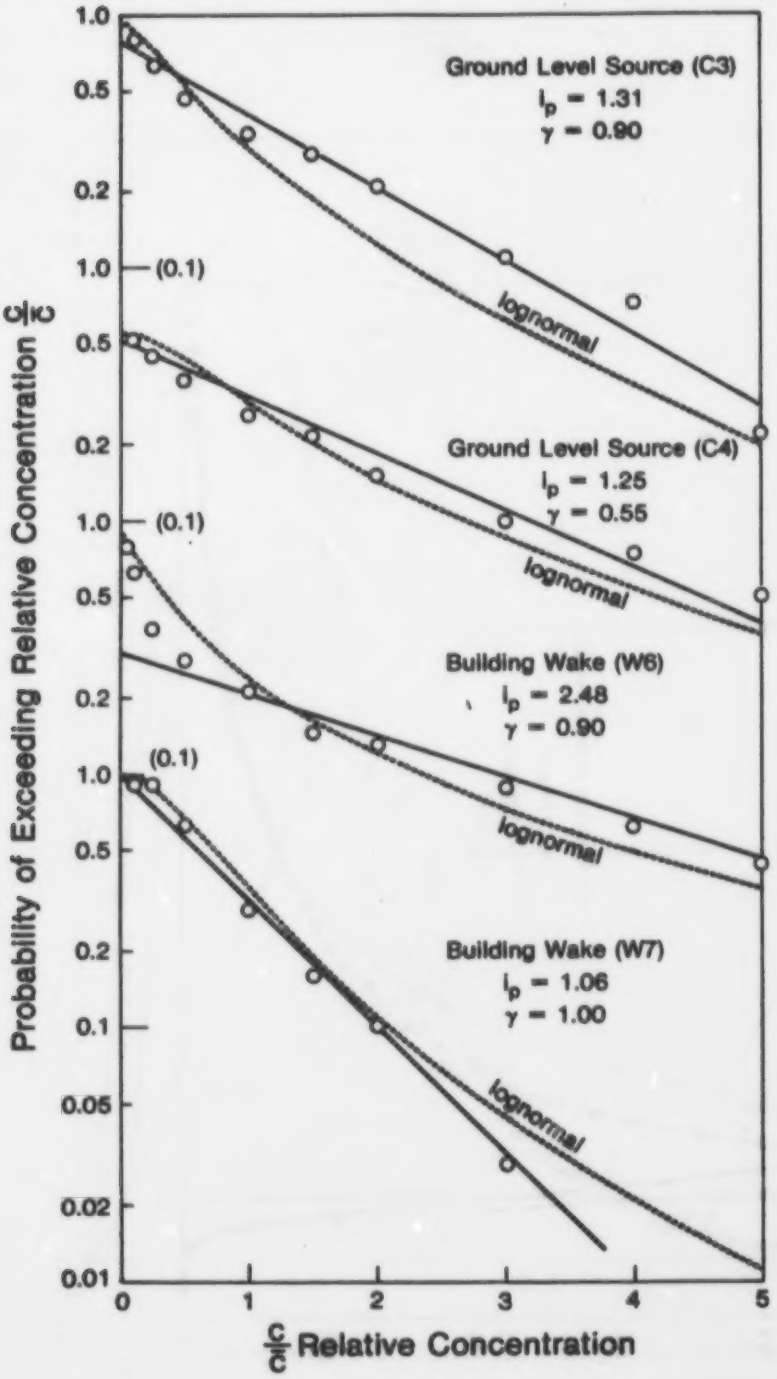

Figure 23. Marginal probability distributions for diffusion from a ground level source in flat terrain and in the wake of a building immediately downwind from the source. Data of Hinds (1969) in Barry (1977).

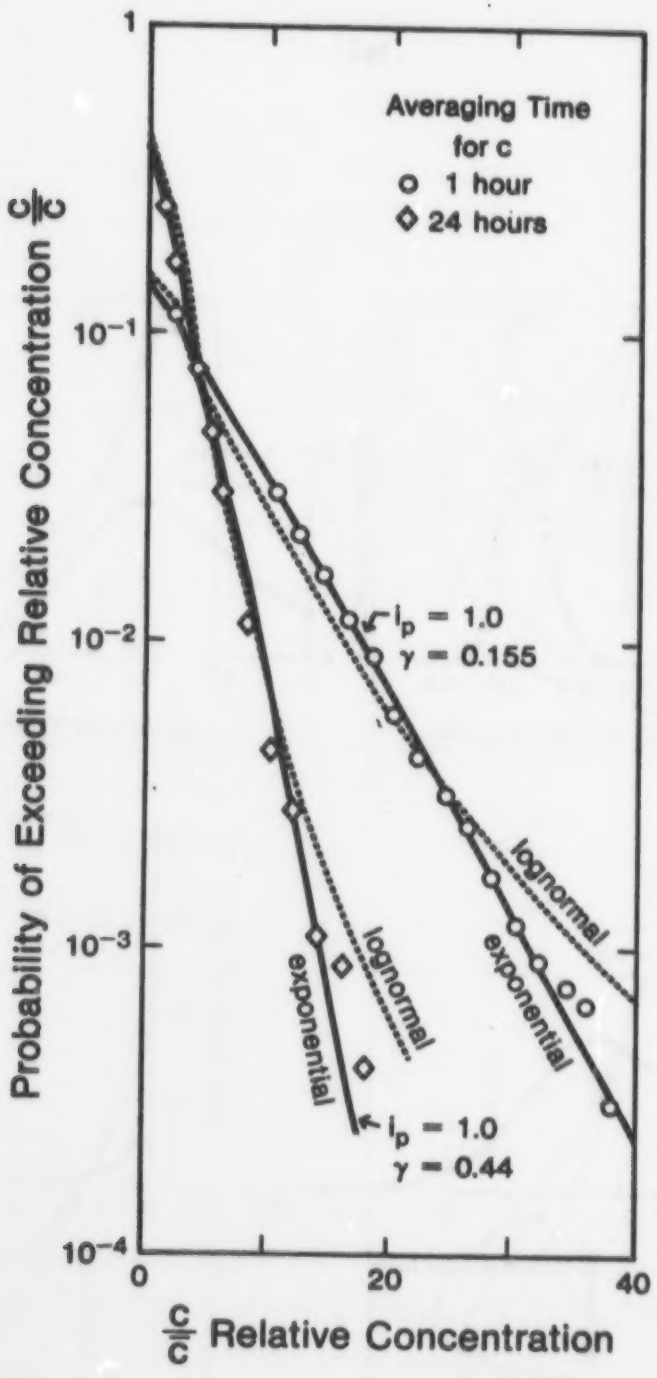

Figure 24. Marginal probability distributions for long time averages at a ground level receptor in an elevated stack plume. Data from Barry (1977), i_p = 1.0 assumed.

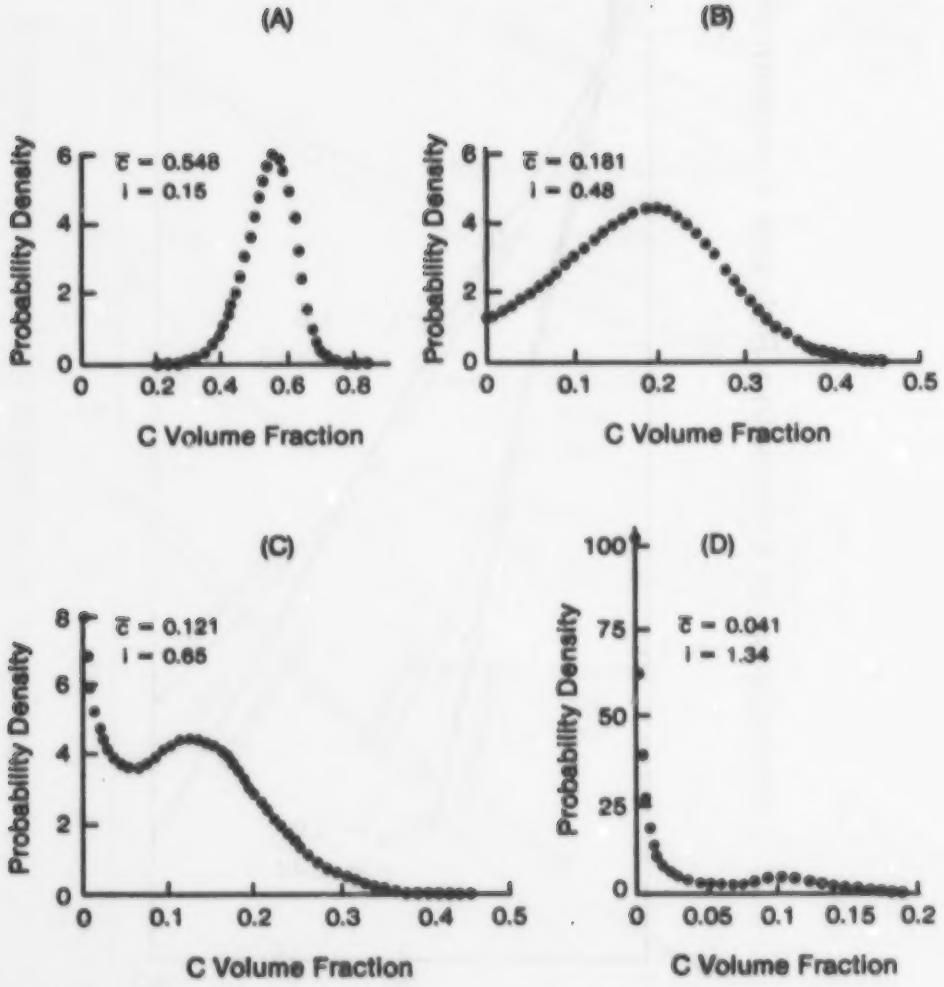

Figure 25. Probability distributions near the nozzle outlet at x/d = 10 in a round methane jet in air.
(A) $r/\sigma_r = 0$, (B) $r/\sigma_r = 1.6$ (C) $r/\sigma_r = 1.8$ (D) $r/\sigma_r = 2.2$
from Birch et. al. (1978).

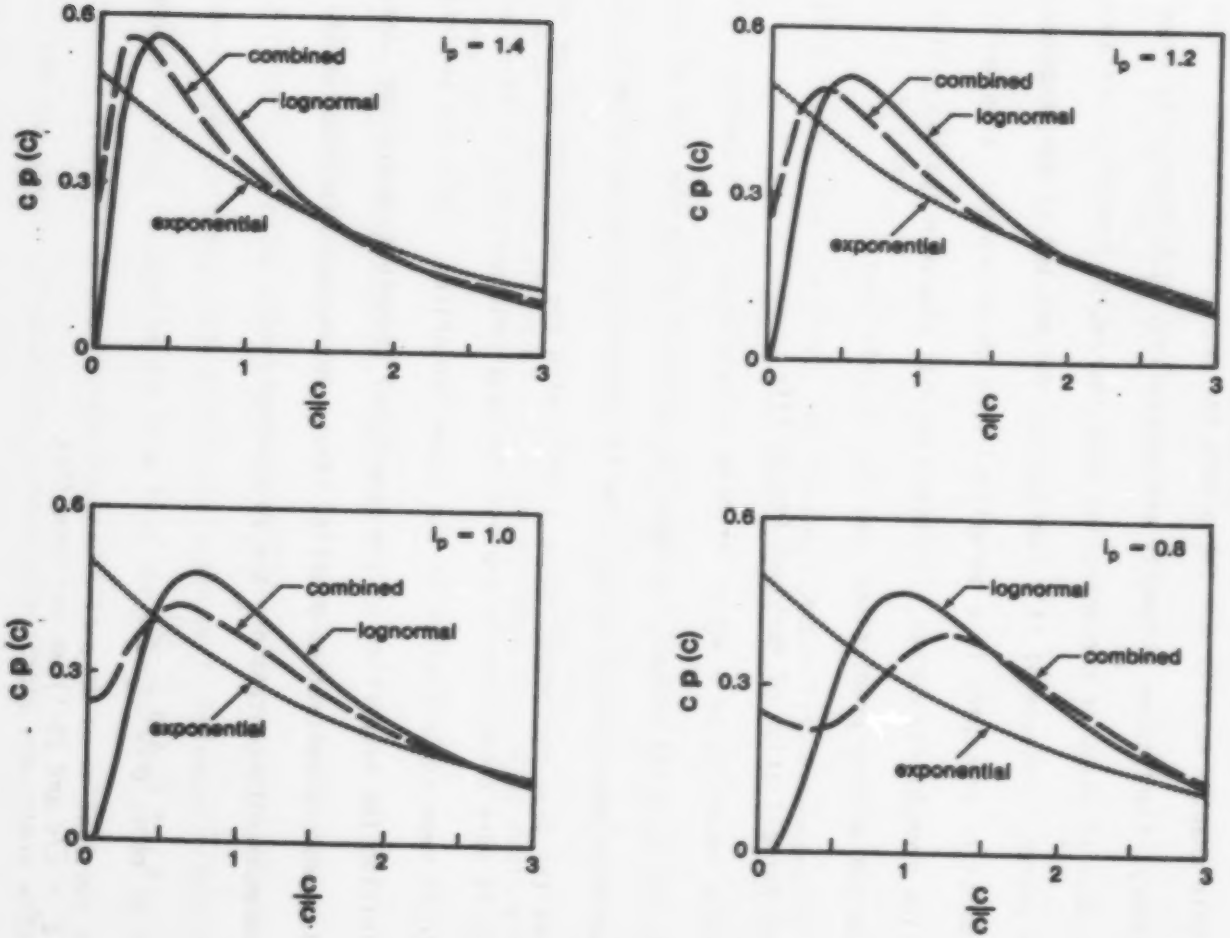

Figure 26. Combined exponential-log normal distributions for plumes with intermittency $\gamma = 0.5$ and varying conditional "plume" fluctuation intensity. Core-bulk structure fractions $(1-\gamma)$ bulk, γ core.

CHAPTER 5

Crosswind Variation of Exposure Risk

In this chapter we will use the models for mean concentration, intermittency, fluctuation intensity and probability distribution to predict the behaviour of peaks of concentration with increasing crosswind distance from the source. Because of it's complexity, the results of the combined models cannot be presented in a normalized form which produce a universal set of risk probability curves. Instead, we will choose a typical full-scale situation and develop predictions specific to this case.

Crosswind Normalization of Marginal Probability

Before embarking on a set of example calculations, it is possible to simplify the marginal probability function which predicts the risk of exceeding some specified peak concentration. The log-normal concentration distribution expresses the peak concentration as a ratio with the conditional plume median c_{mp}. It is more convenient to express the peak concentration in terms of the local total mean concentration on the plume centerline. This can be done by exploiting the properties of the log-normal distribution and the Gaussian crosswind mean concentration profile. For a log-normal distribution the mean and median concentrations are related by

$$c_{mp} = \overline{c}_p \exp\left[-\frac{\sigma_{\ell p}^2}{2}\right] \tag{5-1}$$

using $\overline{c}_p = \overline{c}/\gamma$ and (5-1) we can show that

$$\ell n\left(\frac{c}{c_{mp}}\right) = \ell n\left(\frac{\gamma c}{\overline{c}\exp\left[\frac{-\sigma_{\ell p}^2}{2}\right]}\right) \tag{5-2}$$

Then, the crosswind Gaussian profile for mean concentration relates the local mean, $\bar{c}$, to the value, $\bar{c}_o$, on the plume centerline at y=0 and the same elevation z, from (2-1) and (2-2)

$$\bar{c} = \bar{c}_o \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \tag{5-3}$$

Substituting (5-3) for $\bar{c}$ in (5-2),

$$\ell n\left(\frac{c}{c_{mp}}\right) = \ell n\left(\frac{\gamma c}{\bar{c}_o}\right) + \frac{y^2}{2\sigma_y^2} + \frac{\sigma_{\ell p}^2}{2} \tag{5-4}$$

Finally, using (5-4) in (4-20) the marginal probability of exceeding $c/\bar{c}_o$ is

$$MDF = \gamma\left[1 - \frac{1}{2}\left(1 + \text{erf}\left(\frac{\ell n\left(\frac{\gamma c}{\bar{c}_o}\right) + \frac{y^2}{2\sigma_y^2}2 + \frac{\sigma_{\ell p}^2}{2}}{\sqrt{2}\ \sigma_{\ell p}}\right)\right)\right] \tag{5-5}$$

This form allows the effect of crosswind distance y/σ_y to be dealt with more easily.

Release Conditions for Sample Calculations

Downwind distance from the source is expressed in the plume variance model in terms of plume dimensions $\lambda = \sqrt{\sigma_y\sigma_z}/H$, where H is the atmospheric boundary layer thickness (which sets the scale of turbulence). This plume dimension scaling is used in the hope that the results will apply to all atmospheric stabilities and terrain roughness through their indirect effect on λ. For the sample calculations, a neutrally stable atmosphere with a terrain roughness z_o = 100 cm will be used, with the three minute average plume widths for neutral stability given by

$$\sigma_y = 0.16\, x^{0.88} \tag{5-6}$$

$$\sigma_z = 0.46\, x^{0.68} \tag{5-7}$$

where σ and x are in meters. Taking the neutrally stable boundary layer thickness as H = 650 m we find

$$\lambda = 4.2 \times 10^{-4}\, x^{0.78} \tag{5-8}$$

The two chosen downwind locations are $\lambda = 0.10$ and $\lambda = 0.18$ for an elevated source and $\lambda = 0.18$ for a source at 10 meters. These release conditions and the calculated plume statistics are summarized in Table 5-1.

Calculating Risk Probability

The following step-by-step procedure may be used to compute risk probability.

1. The hazardous peak level "c" is set using toxicological studies
2. Compute the plume centerline mean concentration $\bar{c}_o$ at the receptor height using (2-1), (2-2) and (2-3) with y = 0.
3. Compute the plume fluctuation intensity i_p and the intermittency γ using the step-by-step procedure outlined in Chapter 3.
4. Compute the log standard deviation $\sigma_{\ell p}$ from i_p using (4-16).
5. Finally, using γ, the ratio of $c/\bar{c}_o$, and σ_ℓ, compute the probability of exceeding $c/\bar{c}_o$ from the marginal distribution (5-5) at selected values of crosswind position y/σ_y.

TABLE 5-1

RELEASE CONDITIONS AND PLUME STATISTICS FOR SIMULATION

Source Height h	γ	Downwind Distance x	Plume Fluctuation Intensity i_p	Intermittancy γ_{theory}
10 m	0.18	2370 m	0.41	1.00
65 m	0.10	1110	1.17	0.096
	0.18	2370	0.70	0.712

Other parameters:

surface roughness	Z_o = 100 cm
boundary layer thickness	H = 650 cm
source diameter	d = 2 m
receptor location	Z = 0 (ground level)

The results of these calculations for the ground level source are shown in Figure 27. The theory produces an instability in the outer edges of the plume at $y > 2\sigma_y$ where it unrealistically predicts increasing risk with greater crosswind distance. This instability is caused by the tendency of the theory to underpredict the intermittency, γ, in the fringes of the plume, see Figure 17 for example. With the exception of this minor deficiency, the theory works quite well for marginal probabilities greater than about 1% of the centerline MDF.

The simplest correction is simply to set all probabilities less than 0.01 times the centerline marginal probability equal to zero. Another way of correcting for the error in estimating γ in the plume fringes is to use an additive correction. One that worked well and was simple use is

$$\gamma = \gamma_{theory} + 0.02 \quad \gamma_{theory\ @\ y=0} \tag{5-9}$$

The effect of applying this correction is shown in the lower graph of Figure 27.

The same risk probability distributions for two positions downwind from a high release are shown in Figures 28 and 29. Because the coordinates of Figures 27 through 29 are semi-logarithmic, it can be seen that the exposure risk probability drops very rapidly with distance off the plume center line. What is also apparent is that there seems to be no universal functional form for the risk-distance curves, and further exploration is required to determine values for the integrated risk probability for varying wind direction.

Some Unresolved Difficulties

In developing the models presented in this report, two problems have been ignored. The first is that continuous source data used to formulate an equation for concentration fluctuations cannot help us in estimating these fluctuations in the leading edge in a transient plume. In our model we simply ignore the effect of along-wind diffusion on plume fluctuation intensity and intermittency. Our observations of constant plume fluctuation intensity at fixed downwind plume cross-sections, combined with the rather slow along-wind changes in plume intensity i_p make an assumption of constant plume intensity in the along-wind difussed plume seem reasonable. On the other hand, the plume intermittency would be highly variable in the leading edge of a transient plume. However, it should be kept in mind that the intermittency in the along-wind direction does not directly affect the risk, but simply indicates the variability in arrival time of the plume leading edge.

A second important problem that has simply been ignored is the difficulty in specifying source diameter of a rising momentum jet. The source diameter term, d, is a critical parameter in determining plume intermittency caused by vertical and cross-wind meandering. For a momentum jet, atmospheric turbulence will not be effective in deflecting the plume to cause this meandering until the high speed jet has entrained enough ambient air so that its vertical velocity is of the order of the average windspeed U. Thus, the effective diameter seen by the atmosphere in causing plume meandering is this much larger diameter after the jet slows down by air entrainment. It would seem reasonable to assume that the effective source diameter, d, used to compute variance source strength should be the jet diameter after air entrainment has reduced its upward velocity to a value about equal to the windspeed U.

While each component of the model is firmly based on physical reasoning, the user must be aware that many of the empirical constants are derived for only a single data set, and often from wind tunnel rather than full scale atmospheric measurements. In particular, further studies are required to test the assumption that all atmospheric stability and terrain roughness effects can be accounted for by expressing plume development in terms of a normalized plume size λ.

To be an effective tool in planning and regulation a risk analysis model for toxic gas exposure must be versatile and firmly founded on physical principles. At the same time its principles must be simple enough to be understood by a layman in a court of law. While the computational details of the fluctuating model presented here are often complicated, the physical principles are always simple. Anyone who has watched a smoke plume meander and mix in the atmosphere should be able to understand the concepts behind inhomogeneous mixing, intermittency, and the probability of exposure to peak concentrations.

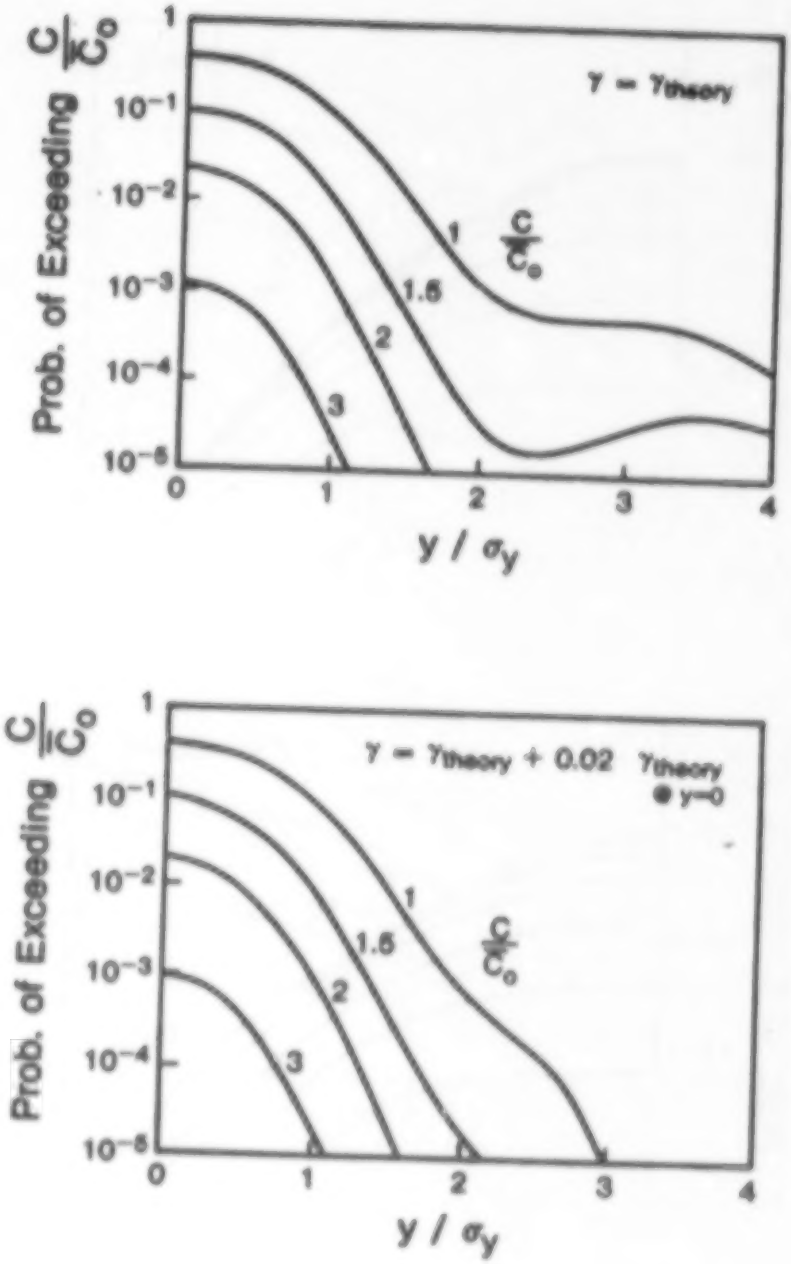

Figure 27. Crosswind variation of exposure risk for various peaks above plume centerline concentration $\bar{C}_o$. Low level source h = 10 m at γ = 0.18. (2370 m downwind).

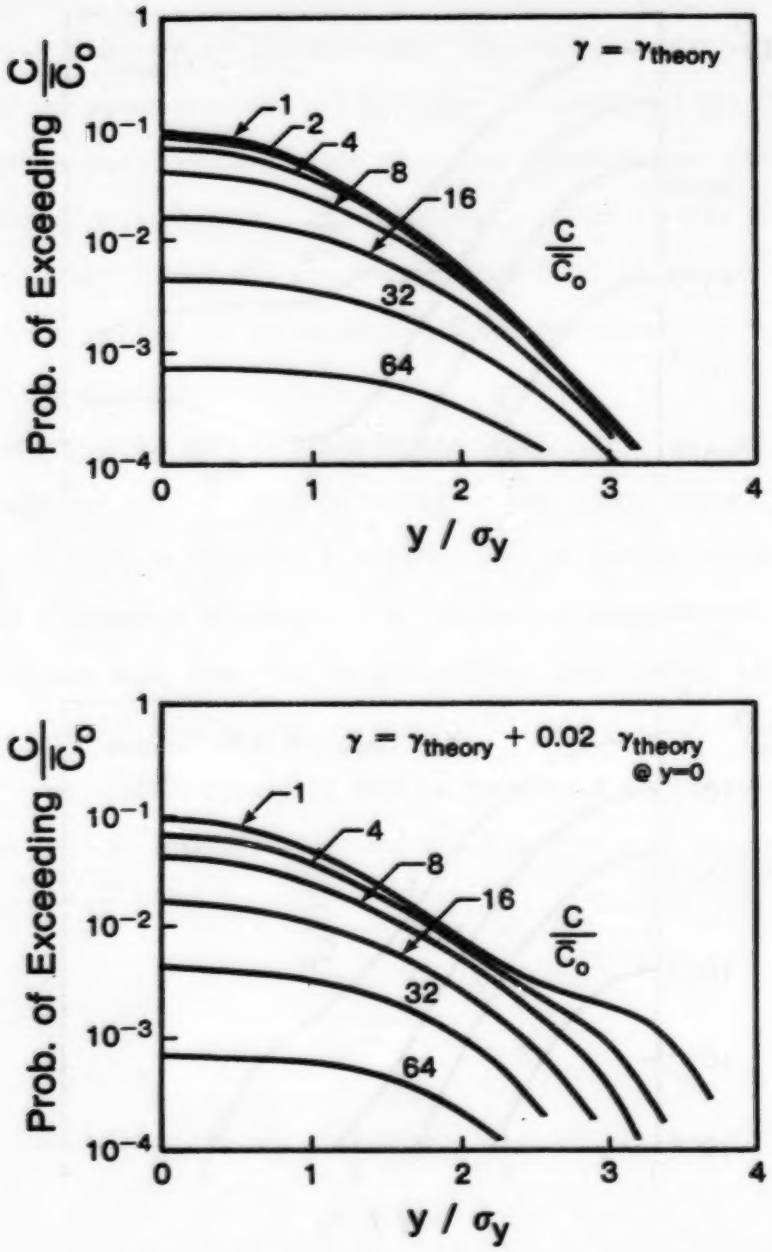

Figure 28. Crosswind variation of exposure risk for various peaks above plume centerline concentration. High source h = 65 m. at λ = 0.10 (1110 m downwind).

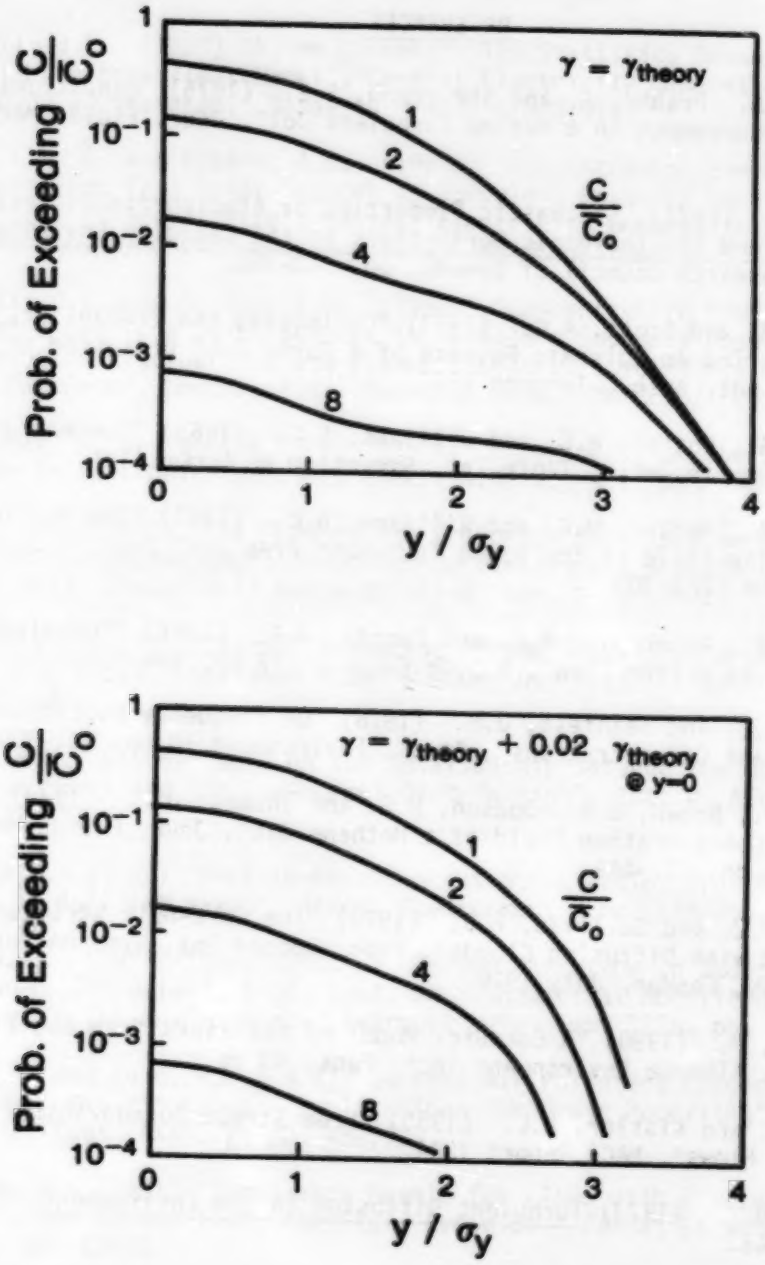

Figure 29. Crosswind variation of exposure risk for various peaks above plume centerline concentration.
High source h = 65 m at λ = 0.18 (2379 m downwind)

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